Different Dynamics for Optimal Association in Heterogeneous Wireless Networks

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EXTENDED ABSTRACT

Most of recent mobile equipment now supports different network technologies (WiFi, WiMax, LTE, Bluetooth and such like). Meanwhile, network operators offer services through these different technologies. The superposition of the different technologies (using different frequency band) increases the potential throughput of the system and hence global performance.

Furthermore, new norms enable mobiles to dynamically switch between these different technologies while maintaining communications. This is known as vertical handover and the consequent network is called heterogeneous. This dynamic switching spares the need of predicting the users’ behaviors to obtain efficient association schemes, opening the way for real-time dynamic association algorithms.

In game theory, the recent evolutionary framework explicitly takes into account the dynamic nature by which individuals learn equilibria. The basic idea in the evolutionary framework is that individuals of populations naturally mimics the behavior of well-fitted elements. Based on the underlying mimics mechanism and on the initial conditions, the overall behavior of a population follows a trajectory of a differential equation, called dynamics.

In this work, we compare two classes of algorithms that approximate several dynamics defined in evolutionary game theory when applied to the user-network association problem in heterogeneous networks. In particular, we study their performance in terms of quality of the obtained equilibria and convergence speed. We further study their complexity and robustness properties with respect with erroneous measurements.

**Keywords**— Distributed Algorithms; Hybrid Wireless Networks; Evolutionary Games; Potential Games, Replicator Dynamics, Vertical Handover; Fairness.

I. CONTEXT AND MODEL

A heterogeneous wireless system consists of a set of overlapping cells of different technologies, as depicted in Fig. 1. The optimal association problem, in a distributed setting can be modeled as a game \((\mathcal{N}, \mathcal{I}, \mathcal{R})\) where \(\mathcal{N}\) is the set of players (mobile equipment), \(\mathcal{I}\) the set of their possible actions (that is to say the choice of the network technology) and \(\mathcal{R}\) their reward function, which can be any performance function, and is taken in the remainder of this paper as their received throughput. In the following, the action taken by a mobile \(n\) is denoted \(i_n\) and her reward is \(r_n\). Additionally, \(\mathcal{I}_n\) denotes the set of choices of player \(n\).

In the general case, the throughput received by a mobile would depend on the actions taken by all other mobiles:

\[
\alpha = \sum_{m \in \mathcal{N}} \text{vect}(i_m, m \in \mathcal{N}) \rangle = r_n(i_n).
\]

As an efficiency measure, we can consider the \(\alpha\)-fair allocation [2], which is, for a given \((\mathcal{N}, \mathcal{I}, \mathcal{R})\), the vector of choices \(I^* = (i_1, i_2, ..., i_n)\) that maximizes

\[
\sum_{n \in \mathcal{N}} G_\alpha(r_n(I)) \quad \text{with} \quad G_\alpha(x) = \frac{x^{1-\alpha}}{1-\alpha}.
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\]

The Nash equilibria of this game are generally inefficient. Yet, it is known that a companion game can be built with

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the same set of users and actions but different reward functions [1]. For an adequate choice of the new reward function, the objective function is a potential (Lyapunov function) of the companion game and hence the Nash equilibria of the newly defined game are local maxima of the objective function. (Note that in the case of arbitrary throughputs, the objective function may not be concave.) Adequate formulations of the rewards in the companion game are (see [1] for details): 
\[ u_n(I) = r_n(I) - \sum_{m \neq n, i_m = i_n} G_\alpha(r_m(I)) - G_\alpha(r_m(l_n - n)). \]
Where \( l_n - n = \text{vect}(p \in \mathcal{N} | i_p = i_n \text{ and } p \neq n) \).

In the following, we focus on dynamics associated to the Nash equilibria of the newly defined game and hence the Nash equilibria of the companion game.

II. DIFFERENT TYPES OF DYNAMICS

We have seen that the \( \alpha \)-fair performance measure is a potential function for the companion game, and hence that the Nash equilibria have good optimality properties. The question is now of how to obtain and compute such equilibria. The evolutionary game framework [2] provides some dynamics that model the convergence process under several forms. Three major dynamics were defined, namely the replicator dynamics, the best-response dynamics and the projection dynamics.

A. Replicator Dynamics

\[ \mathcal{T}_n(q) = \sum_{i \in \mathcal{I}_n} q_{n,i}f_{n,i}(q) \]
For all \( n \in \mathcal{N}, \forall i \in \mathcal{I}, \frac{dq_{n,i}}{dt}(q) = q_{n,i} \left( f_{n,i}(q) - \mathcal{T}_n(q) \right). \]

B. Best Response Dynamics

Best response dynamics represents a class of strategy updating rules, where players strategies in the next round are determined by their best responses to some subset of the population. Importantly, in these models players only choose the best response on the next round that would give them the highest payoff on the next round. Players do not consider the effect that choosing a strategy on the next round would have on future play in the game. This constraint results in the dynamical rule often being called myopic best response.

C. Projection Dynamics

\[ \hat{f}_n(q) = \frac{1}{|\mathcal{I}_n|} \sum_{i \in \mathcal{I}_n} f_{n,i}(q) \]
For all \( n \in \mathcal{N}, \forall i \in \mathcal{I}, \frac{dq_{n,i}}{dt}(q) = q_{n,i} \left( f_{n,i}(q) - \hat{f}_n(q) \right). \]

D. Price of Anarchy

The price of anarchy [1] is defined as the ratio of the worse Nash equilibria over the maximum of the considered performance measure, here Eq. 1. For a given instance of the problem \((\mathcal{N}, \mathcal{I}, \mathcal{P})\), let \( \mathcal{I}^{NE} \) be the set of Nash equilibria. Then, the price of anarchy is:
\[ \text{PoA} = \inf_{\mathcal{N}, \mathcal{I}, \mathcal{P}} \frac{\inf_{\mathcal{I}^{NE}} \sum_{n \in \mathcal{N}} G_\alpha(r_n(I))}{\sup_{\mathcal{I}} \sum_{n \in \mathcal{N}} G_\alpha(r_n(I))}. \]

III. ALGORITHMS

Once a dynamic has been chosen, the next step is to design an algorithmic implementation of the dynamics that will be used by the players to define their strategies.

A. Gradient Descent Based Algorithms

A first possible implementation is a discretization of the dynamics, based on a gradient descent, inspired from a numerical Runge-Kutta integration of differential equations. For the replicator dynamics, the first order integration yields
\[ q_{n,i}(t + 1) = q_{n,i}(t) + \beta q_{n,i} \left( f_{n,i}(q) - \mathcal{T}_n(q) \right). \]

The case of the projection dynamics being similar.
\[ q_{n,i}(t + 1) = q_{n,i}(t) + \beta q_{n,i} \left( f_{n,i}(q) - \hat{f}_n(q) \right). \]

This method has been used in [3]–[5] for routing problems. In our context however, such a discretization implies that each user must probe all the cells she can connect to, in order to compute an approximation of the gardient on the dynamics, at each time step. There exists an alternative to this by only probing one cell at each iteration, chosen randomly among all cells, according to a distribution proportional to the state. This new discretization technique can be seen as a stochastic approximation of the dynamics [6].

B. Stochastic Approximation Based Algorithms

The use of stochastic approximations as a noisy discretization of differential dynamics systems has been proven effective on the context of communication distributed system ([7]). This is because it directly translates into on-line algorithms that are both distributed and adaptive and that they can be proved to converge to the set of stable points of the dynamics [7], [8]

As for the replicator dynamics, the stochastic approximation iteration is of the form
\[ q_{n,i}(t + 1) = q_{n,i}(t) + \beta r_n(\ell^n(s)) (1_{s_n = i} - q_{n,i}(t)), \]
where \( s_n \) is a random variable such that \( P(s_n = 1) = q_i \) and \( \beta \) is the step size. The approximation equation for the projection dynamics is similar:
\[ q_{n,i}(t + 1) = q_{n,i}(t) + \beta r_n(\ell^n(s)) (1_{c_n = i} - q_{n,i}(t)). \]

Here \( c_n \) is a random variable uniformly distributed among the cells for which \( q_{n,i} \neq 0 \).

IV. PROPERTIES

This section is devoted to the comparison of the different methods, i.e. the couples (dynamics , approximation algorithm), in terms of speed and robustness.
A. Convergence Speed

A first important feature is to evaluate the number of iterations used in the algorithm before an equilibrium point of the dynamics is reached. This evaluation is based on an analytical study when the dynamics admits a Lyapounov function and its decreasing rate can be computed. It is also based on numerical experiments. We also show that in all cases, the choice of the discretization steps can help to decrease the number of iteration before convergence. For instance, the adaptation of the step size to the current drift improves the convergence speed dramatically.

B. Robustness with respect to Noise on the Measures

Another desirable property of the implementation is that it is robust to perturbations on (the measure of) the variables used at each iteration of the algorithm. In that respect, it should be clear that algorithms based on stochastic approximations are better adapted to random noise and are resilient to perturbations in terms of the quality of the equilibrium as well as the speed of convergence.

References