Information Theoretic Congestion Games in Heterogeneous Wireless Networks

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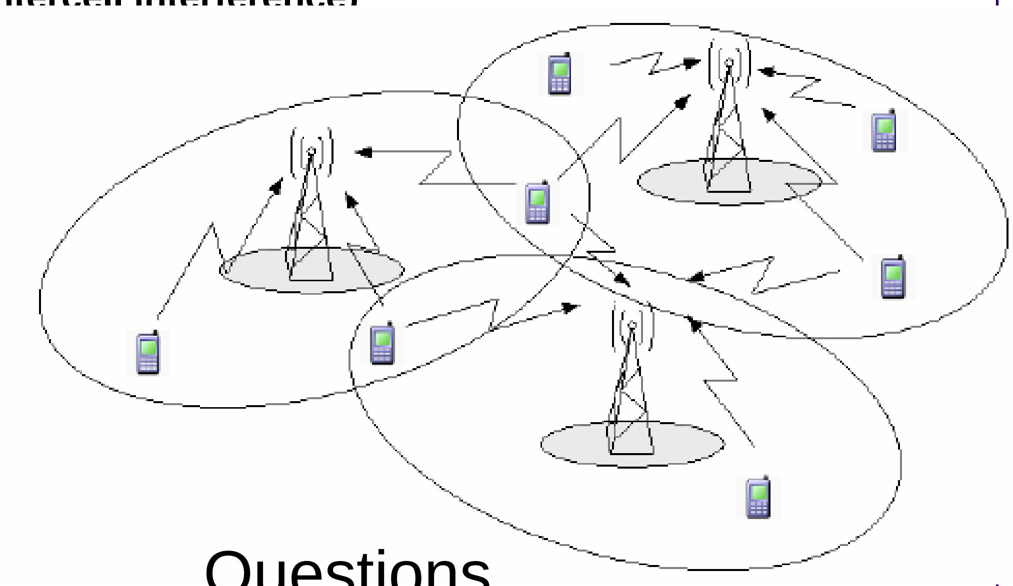
Introduction (1/2)

➔ Definition 1 (Heterogeneous networks, HN)

- A group of K mobile terminals
  - Each of them is equipped with a cognitive radio [Mitola-Phd-2000][Fette-Book-2006]
- A set of S base stations
  - Connected through perfect links
  - Non-overlapping frequency bands (no intercell interference)
  - Bandwidths: $B_1, ..., B_S$
  - Noise levels: $N_1, ..., N_S$
  - Number of antennas:
    $n_{t,1}, \ldots, n_{t,K}$
    $n_{r,1}, \ldots, n_{r,S}$
  - Ex ample of HN: 802.11xx, GPRS, UMTS
  - A key idea: from the information theoretic standpoint for large systems: A technology is characterized by a few simple parameters [Lasaulce-Valuetools-2007]



Questions

Introduction (2/2)

Definition 2 (Shannon capacity)

$$X \longrightarrow Y = X + Z$$

$$X \in \mathbb{R}$$

$$C = \frac{1}{2}\log_2\left[1 + \frac{E(X^2)}{E(Z^2)}\right]$$

P

N

$$\begin{cases} p_{XY}(x,z) = p_X(x)p_Z(z) \\ Z \sim \mathcal{N}\left(0, E(Z^2)\right) \\ p_{\underline{Z}}(z_1,\ldots,z_n) = \prod_{i=1}^{n} p_Z(z_i) \\ P_e^{(n)} = \epsilon > 0 \end{cases}$$

- **Influential physical parameters**
  - **Noise level: "noise ↓" ⇒ "capacity ↑" [Shannon-BSTJ-1948]**
  - **Signal bandwidth: "B ↑" ⇒ "capacity ↑" [Shannon-BSTJ-1948]**
  - **Number of dimensions (receive antennas): "n ↑" ⇒ "capacity →↑" [Telatar-ETT-1999][Tse-Book-2005]**
  - **...**

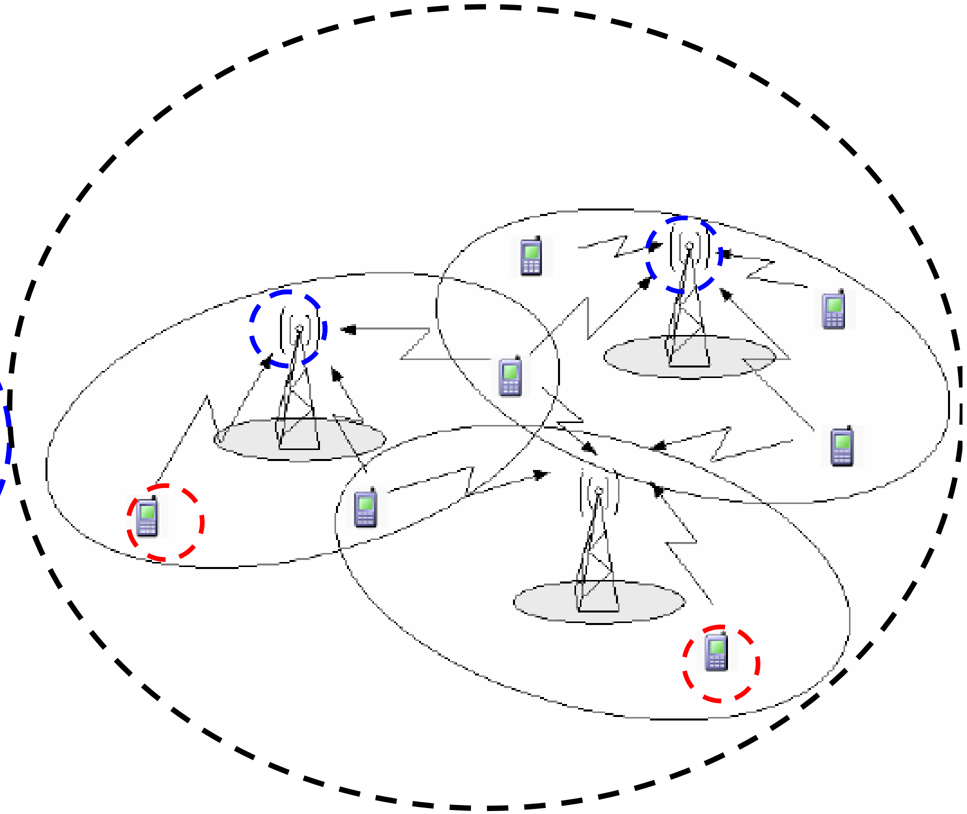
Problem statement (1/2)

Network features

- Decentralized
- Users are selfish and want to maximize their transmission rate
- Signal model (baseband received signal @ BS s):

$$\underline{Y}_s(\tau) = \sum_{k=1}^{K} \mathbf{H}_{s,k}\underline{X}_{s,k}(\tau) + \underline{Z}_s(\tau)$$

- Receiver: single-user decoding (SUD) xor successive interference cancellation (SIC)
- Good network sum-rate efficiency



Problem statement (2/2)

➔ **Definition 3 (game under investigation)**

- **Players: mobile terminals**
- **User's strategy: power allocation (PA) between the S available systems**

$$\forall k \in \{1, ..., K\},\ \mathbf{Q}_k = \left[\mathbb{E}\left(\underline{X}_{1,k}\underline{X}_{1,k}^H\right) ... \mathbb{E}\left(\underline{X}_{S,k}\underline{X}_{S,k}^H\right)\right]$$

- **User's utility: individual Shannon transmission rate**

$$u_k^{(D)}(\mathbf{Q}_1, ..., \mathbf{Q}_K) = \left| \begin{array}{ll} \sum_{s=1}^{S} I(\underline{X}_k;\underline{Y}_s) & \text{if D} \equiv \text{SUD} \\ \sum_{s=1}^{S}\sum_{i=1}^{K} p_{s,k}^{(i)} I(\underline{X}_k;\underline{Y}_s|\underline{X}_s^{(i)}) & \text{if D} \equiv \text{SIC} \end{array} \right.$$

- **Set of strategies:** $\mathbf{Q}_{s,k} \geq 0,\ \sum_{s=1}^{S} \mathrm{Tr}\left(\mathbf{Q}_{s,k}\right) \leq P_k$ → Hard handover (HH) / Soft handover (SH)

→ Equilibrium: existence, uniqueness, determination (optimum PA), social efficiency.

Technical Background

Closest works

- [Lasaulce-Valuetools-2007] ← CDMA MIMO OFDM
  - Heterogeneous networks
  - Centralized case
- [Liang-WCNC-April2008]
  - Downlink traffic + HH
  - Single-user receiver (SUD)
  - Single-antenna terminals
- [Mertikopoulos-WNC3-April2008]
  - Their analysis holds only for 2 BSs + HH
  - Specific utilities: round robin protocol (no multiuser interference)
  - Convergence analysis (replic. dynamics)
  - Single-antenna terminals
- [Altman-Physcomnet-April2008]
  - Static channel
  - Single-antenna terminals
  - Focus on the Braess paradox
  - Single-antenna terminals

What we present here

- [Belmega-ISCCSP-March2008]
  - Uplink case
  - Static case with single-antenna terminals
  - Hard and <u>soft</u> handovers
  - SUD and <u>SIC-based receivers</u>
- Extensions
  - More on hard soft handovers
  - <u>Social efficiency</u> analysis
  - Case of <u>multi-antenna</u> terminals: treated thanks to random matrix theory

Single-antenna terminals & Static channels

Signal model for the case of single-antenna terminals

Baseband signal received at BS s

static channel gains

$$y_s = \sum_{k=1}^{K_s} h_{s,k} x_{s,k} + z_s,$$

Assumption
$B_1 = \ldots = B_S$

- z_s complex white Gaussian noise $\mathcal{N}(0, N_s)$;
- The total number of users is $K = \sum_{s=1}^{S} K_s$. (HH case)
- $\mathbb{E}|x_{s,k}|^2 = \alpha_{s,k} P$ with $\sum_{s=1}^{S} \alpha_{s,k} = 1$
  - Soft handover: $\alpha_{s,k} \in [0, 1]$;
  - Hard handover: $\alpha_{s,k} = 0$ or $\alpha_{s,k} = 1$;
- BS use either Single User Decoding (SUD) or Successive Interference Cancellation (SIC).

arbitrary number of users

Soft handover with single-user decoding

$\underline{\alpha} = (\underline{\alpha}_1, ..., \underline{\alpha}_K)$ and $\underline{\alpha}_k = (\alpha_{1,k} ... \alpha_{S,k})$

- The payoff of user k

$$\eta_{s,k} = \frac{|h_{s,k}|^2 P_k}{N_s}$$

$$u_k(\underline{\alpha}) = \sum_{s=1}^{S} \log_2 \left[1 + \frac{\alpha_{s,k}\eta_{s,k}}{1 + \sum_{\ell \neq k} \alpha_{s,\ell}\eta_{s,\ell}} \right]$$

- Each user wants to selfishly maximize its own payoff knowing that the other users in the network will act in the same way
- Existence and Uniqueness of an NE [Rosen-Econometrica-1965]
- The NE will be the solution to $\frac{\partial \mathcal{L}_k}{\partial \alpha_{s,k}}(\underline{\alpha}) = 0, \forall k$ with

$$\mathcal{L}_k(\underline{\alpha}, \{\lambda_k\}_{k \in \mathcal{K}}) = -u_k(\underline{\alpha}) + \lambda_k \left(\sum_{s=1}^{S} \alpha_{s,k} - 1 \right)$$

Soft handover with successive interference cancelation

Coordination mechanism:
S orthogonal, equiprobable and discrete signals

- The payoff of user k

$$u_k(\underline{\alpha}) = \sum_{s=1}^{S} \frac{1}{K} \sum_{i=1}^{K} \left[\frac{1}{\binom{K-1}{i-1}} \sum_{J_k^{(i-1)}} \log_2 \left(1 + \frac{\eta_{s,k}\alpha_{s,k}}{1 + \sum_{\ell \in J_k^{(i-1)}} \eta_{s,l}\alpha_{s,\ell}} \right) \right]$$

where $J_k^{(i-1)} = \{I \subset \mathcal{K} \setminus \{k\} |$ such that: $|I| = i - 1\}$.

- The NE will be the solution to $\frac{\partial \mathcal{L}_k}{\partial \alpha_{s,k}}(\underline{\alpha}) = 0, \forall k$ with

$$\mathcal{L}_k(\underline{\alpha}, \{\lambda_k\}_{k \in \mathcal{K}}) = -u_k(\underline{\alpha}) + \lambda_k \left(\sum_{s=1}^{S} \alpha_{s,k} - 1 \right)$$

Social efficiency of the Nash equilibrium for soft handovers

Social efficiency

- Measured in terms of price of anarchy, POA [Papadimitriou-STC-2001]
- In our case: utility = transmission rate
- POA = sum of utilities optimized jointly/sum of utilities at the NE

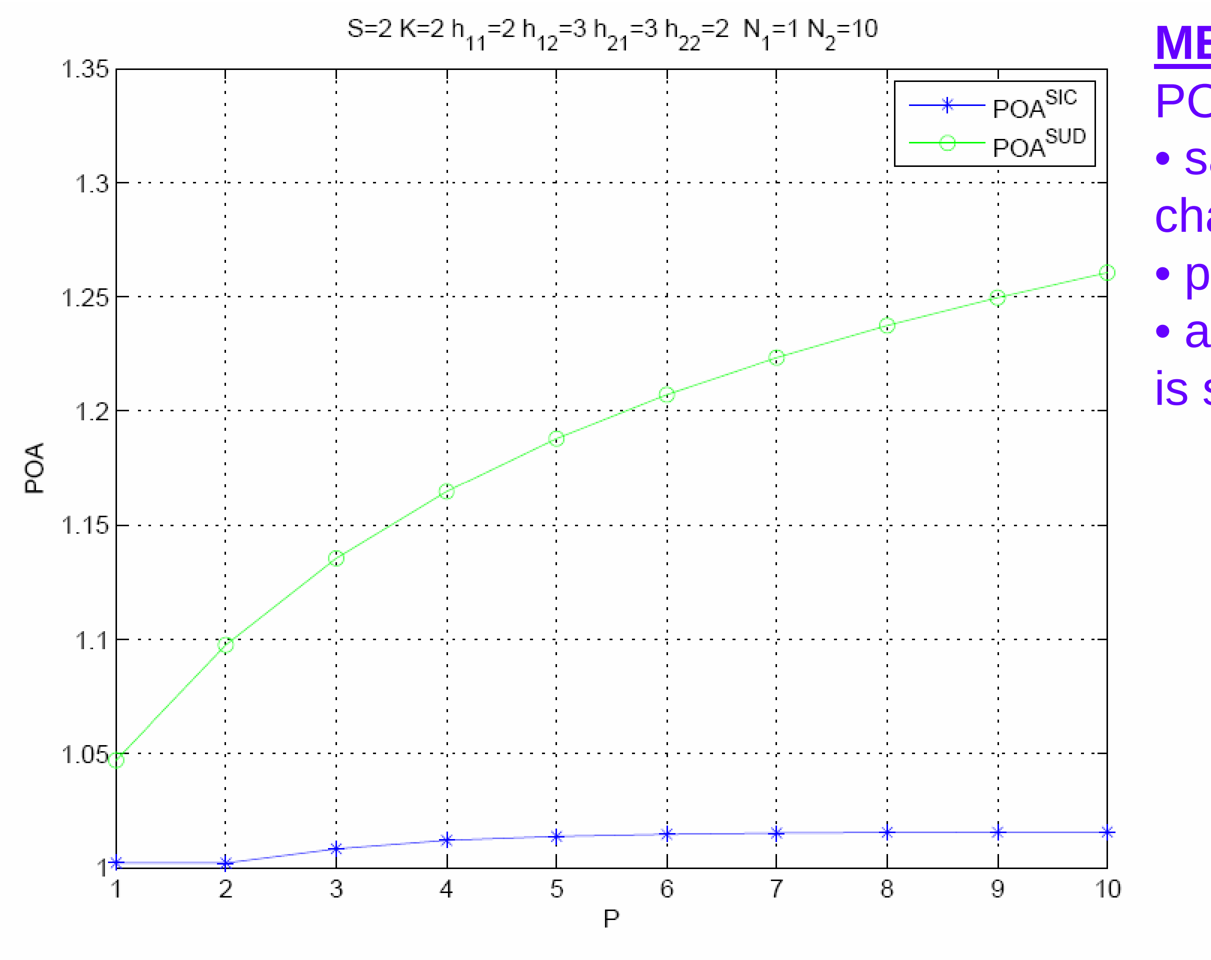
Single-user decoding

- Sum-rate neither convex neither concave w.r.t. $\underline{\alpha}$
- Optimization: exhaustive search
- POA > 1

Successive interference cancelation

- Conjecture: we always have $1 \leq POA \leq 2$
- Result proven in the case where the channel gains are identical
- Simulations show that, expect for special regimes, the POA is often close to 1 →

Simulation example



MESSAGE

POA ≈ 1:

- same observation for fading MIMO channels (Cf cooperative systems)
- pricing for SIC = useless
- although the coordination mechanism is suboptimal

Case of hard handovers (1/2)

- **General comments**
  - Is HH merely a special case of SH? No. E.g. look at the properties of the set of strategies
  - For a finite group of users, there is no equilibrium in general. The special case of a network with 2 users and 2 base stations has been treated by [Altman-WNC3-April2008]: coordination games.

- **Case of large groups of users (populations)**
  - The problem becomes a congestion game
  - Equilibrium: existence (insured), uniqueness (not insured in general), convergence, see [Rosenthal-IJGT-1973][Monderer-GEB-1996][Milchtaich-GEB-1996]

Case of hard handovers (2/2)

- **A couple of results for a large group of users**
  - **Here we make a simplifying assumption: symmetric network in terms of gains**
  - **Single-user decoding**

$$x_s^{NE,SUD} = \frac{1}{S} + \frac{1}{S}\sum_{j=1}^{S} \frac{N_j - N_s}{KP}$$

Rewriting it as a water-filling

$$x_j^{*,C} = \frac{1}{K}\left[\omega - \frac{N_j}{P}\right]^+$$

  - **Successive interference cancelation**

$$u_{s,k}^{(SIC)}(\underline{x}) = \frac{1}{Kx_s}\log_2\left[1 + \frac{Kx_sP}{N_s}\right]$$

$$\begin{cases} \frac{1}{x_s}\log_2\left[1+\frac{Kx_sP}{N_s}\right] = \frac{1}{x_j}\log_2\left[1+\frac{Kx_jP}{N_j}\right] & \forall s,j \in \mathcal{S}, j \neq s \\ \sum_{s=1}^{S} x_s = 1 \end{cases}$$

Social efficiency of the Wardrop equilibrium for HH

- **In terms of POA**
  - **If the repartition is uniform, POA = 1**
  - **For the regime of low signal-to-noise ratios (e.g. spread spectrum systems), POA > 1. Introducing a pricing mechanism makes sense.**

Pricing mechanism

➔ Problems

- Shannon transmission rates are not toys
- We want to maximize the rate but minimize a price

➔ Solution

- Convert the transmission rate into a connection time [Li-TWC-2002]
- In our setup we get
  - The time that user $k \in \{1, ..., K\}$ spends to transmit its data if it is connected to BS $s \in \mathcal{S}$ is:

$$\tau_{s,k}^{(D)}(\underline{x}) = \frac{n_k}{u_{s,k}^{(D)}(\underline{x})}$$

  - **Cost:** $c_{s,k}(\underline{x}) = p(\tau_{s,k}^{(D)}(\underline{x})) + \beta_s$
  - **Spontaneous equilibrium → stimulated equilibrium:** $\beta_s = -p(\tau_{s,k}^{(D)}(x_s^{M-NE,D})) + p(\tau_{s,k}^{(D)}(x_s^{NE,D}))$

Multi-antenna terminals & Fast fading channels

Main differences between static SISO and fading MIMO channels

- **Many channels + they vary over time**
  - **Problem: knowledge of channels gains for large groups of users…**
  - **Example: single-user decoding**

$$u_k^{(s)}(\mathbf{Q}_k, \mathbf{Q}_{-k}) = \frac{1}{n_{r,s}} \left[\log_2 \left| \rho_s + \sum_{\ell=1}^{K} \mathbf{H}_{s,\ell} \mathbf{Q}_{s,\ell} \mathbf{H}_{s,\ell}^H \right| - \log_2 \left| \rho_s + \sum_{\ell=1, \ell \neq k}^{K} \mathbf{H}_{s,\ell} \mathbf{Q}_{s,\ell} \mathbf{H}_{s,\ell}^H \right| \right]$$

  - **… but there is an averaging effect, which can be analyzed thanks to random matrix theory**

    Predictability: good for games
  - **In large decentralized systems (MIMO, CDMA, OFDM, …) the assumption of games with complete information makes sense**
  - **Channel hardening effect in MIMO systems:**
    - **Positive consequence 1: users are indifferent to fading, they only care about its statistics and the base station noise levels**
    - **Positive consequence 2: the proposed (a priori) suboptimal coordination mechanism performs well in terms of sum-rate**

Other issues currently under investigation

- **Clean equilibrium analysis in MIMO systems**
  - Existence and uniqueness of an NE
  - Essentially matrix algebra problems
  - Optimum power allocation
- **Deriving a tight upper bound on the POA in MIMO systems**
  - Especially SIC
- **Treating the case of non-equal bandwidths**
- **Adding spatial considerations in the analysis**
  - Path loss model
- **Considering the effect of the inter-cell interference**
  - The base stations are using the same bandwidth (CDMA systems)
- **Analyzing wireless networks with a small number of users**
  - How to insure the existence of an equilibrium (games with incomplete information?)
- **Dynamics of the network (evolutionary games,…)**