Equitable Multi-Objective Optimization

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based on joint work with
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In many situations, fairness is not equal division.

"Cake" gets bigger

oranges [Fisher, 1981]

two halves

peel and juice

[www.pachd.com]
However, sometimes after an “equal” division the “cake” gets smaller.

Income inequality vs GDP in Poland

GDP per capita, $ (prices from 2004)

Gini index
higher -> less equal

most equal distribution

economic crisis

communism
-> capitalism
transition
IDEA: EXTEND MULTI-OBJECTIVE OPTIMIZATION TO INCLUDE FAIRNESS.
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Multi-objective optimization problem with multiple agents that must be treated fairly.

Centralized decision maker
Agents’ utilities depend on each other (shared system)
Payoffs not transferable

\[
\min (f_1(x), f_2(x), \ldots, f_n(x))
\]

- \(f_1(x)\) – first agent’s objective
- \(f_2(x)\) – second agent’s objective
- \(f_n(x)\) – last agent’s objective

Examples:
- multi-agent scheduling [Agnetis et al, 2004]
- robust shortest path [Perny et al, 2006]
- budget redistribution [Kostreva et al, 2004]
- facility location [Marsch, 1994]
(Values of) agents' utilities have to be directly comparable.

**fair division**
- finite goods to distribute
- each agent: max her share

**cardinal utilities**
- utility functions express strength of preferences

- facility location
  - [Marsch, 1994]
- budget redistribution
  - [Kostreva et al, 2004]
- robust shortest path
  - [Perny et al, 2006]
- multi-agent scheduling
  - [Agnetis et al, 2004]
ECONOMICS AND SOCIAL SCIENCE HAVE SIMILAR PROBLEM: HOW TO MEASURE FAIRNESS OF INCOME DISTRIBUTION

Sort outcomes from worst to best; measure cumulative outcome of n worst outcomes

Lorentz Curve [Lorentz 1905]
Gini index expresses the unfairness in Lorentz curve.

[Gini, 1912]
Gini Index can favor inefficient solutions.

Economic crisis:
- Average salary: $20/month
- Minimum salary: $200/month

End of first boom.
Idea: compare equity of solutions that have the same global efficiency

- economics: Pigou-Dalton principle of transfers
- mathematics: majorization

multi-agent minimization

\[ y = [y_1, y_2, \ldots, y', \ldots, y'', \ldots, y_n] \]
\[ y' > y'' \text{ (y' is worse)} \]

\[ y + \varepsilon'' - \varepsilon' \text{ preferred to } y \]

\[ \varepsilon' = [0, \ldots, 0, \ldots, \varepsilon, \ldots, 0, \ldots, 0] \]
\[ \varepsilon'' = [0, \ldots, 0, \ldots, 0, \ldots, 0, \ldots, \varepsilon, \ldots, 0] \]

e.g. [9,6] is preferred to [10, 5]
both [9,6] and [10,5] have the same global efficiency 15
Axiomatic theory of fairness: properties of a fair division

Axioms:
- Symmetry
- Optimality
- Principle of transfers

lazy workers
(minimizing their work)

number of hours: common valuation

work to be done
A fair solution is symmetric: no one is favored as fair as fair

2
3

1 1

1

===

1

1

1

1

1

3

2

3
A fair solution is optimal: it can’t be improved unilaterally.

fairer than
Principle of transfers:
Rob the riches, feed the poor

fairer than
We define a relation of equitable dominance that fulfills these axioms

Relation $\leq_L$ : partial order on $\mathbb{R}^N$
fulfils axioms of equity

Which is more equitable: [4,6] or [10,2]?

- $[10, 2] =_L [2, 10]$ (symmetry)
- $[2, 10] <_L [4,8]$ (transfers)
- $[4, 8] <_L [4, 6]$ (Pareto-optimality)
- So, [10, 2] less equitable than [4, 6]
These criteria can be encapsulated in a relation of Generalized Lorentz Dominance [Chong, 1976]

(3 agents minimizing outcomes)

- sort outcomes from the worst to the best
  
  $[1, 5, 3]$ -> $[5, 3, 1]$  
  $[2, 5, 2]$ -> $[5, 2, 2]$  

- construct cumulative outcome vector: 
  [worst outcome, sum of two worst outcomes, ..., some of n worst outcomes] 

  $[5, 3, 1]$ -> $[5, 8, 9]$  
  $[5, 2, 2]$ -> $[5, 7, 9]$  

- compare cumulative vectors by Pareto dominance 

  $[5, 7, 9]$ dominates $[5, 8, 9]$
Lorentz dominance gives us straightforward interpretation.

\[ f_1(x) \]

\[ f_2(x) \]

2 criteria minimization

equally better
equally equal

equally worse

pareto-better

pareto-worse

eq worse
The same results can be obtained by using axioms directly

\[ f_1(x) \]

\[ f_2(x) \]


\[ C[100,1] < [99,2] < A[6,2] \]

\[ D[7,0] < [6,1] < [5,2] < [4,3] \]

\[ A[6,2] < [4,4] < E[4,3] \]

\[ F[5,4] > [6,3] \]

\[ A[6,2] < [5,3] < [4,4] \]

Types of dominance:

- \(<\) pareto
- \(<\) transfers
- \(<\) equitable

\[ A < E \]

\[ C < A \]

(None is pareto-dominated by [6,2])

(None pareto-dominance)
Finding equitably-optimal solutions
EXAMPLE APPLICATION: SCHEDULING PROBLEM WITH MULTIPLE AGENTS

Agnetis et al., 2004

One processor
Two sets of jobs

1 || \[ \sum C^G_i, \sum C^R_i \]

green optimization goal: \[ \sum C^G_i = 1 + 6 + 18 = 25 \]
red optimization goal: \[ \sum C^R_i = 3 + 13 = 26 \]

budget based approach: NP-hard
Pareto-set can have exponential size
Multicriteria optimization can model fairness between multiple users.

(best total completion time)

\[
\sum C_i^g = 1 + 2 + 3 \quad \sum t_i = 6 \quad \sum C_i^g = 1 + 2 + 3 + 6 = 12
\]

(better for red agent)

\[
\sum C_i^g = 4 + 5 + 6 \quad \sum t_i = 3 \quad \sum C_i^g = 3 + 4 + 5 + 6 = 18
\]
One Fair Solution
Ordered Weighted Average (OWA) with positive, strictly decreasing weights gives an equitable solution. [Ogryczak, 2000]

- OWA [Yager, 1988]: sort outcomes by increasing performance, then apply weighted average
  
  vector: [1, 10, 5]
  weights: [0.5, 0.2, 0.1]
  
  $\text{OWA} = 0.5 \times 10 + 0.2 \times 5 + 0.1 \times 1$

$\text{OWA}(y') < \text{OWA}(y'') \iff y' \text{ equitably-dominates } y''$

Sketch of proof for $'\leq'$

- symmetry holds: OWA sorts outcomes
- optimality holds: one outcome decreased $\rightarrow$ OWA decreased
- transfers holds: larger outcomes have larger weights
Other equitable aggregation functions are also possible. [Kostreva et al, 2004]

- Aggregator function $g(y)$ maps outcome vector to $\mathbb{R}$
  \[ \min(g(f(x))) \]

Requirements on $g(y)$:
- strictly increasing ($\rightarrow$ on each outcome)
- impartial ($\rightarrow$ not sensitive to permutations of $y$)
- strictly Schur-convexity: $g(y_1, \ldots, y_i - \varepsilon, \ldots, y_i'' + \varepsilon, \ldots, y_m) \leq g(y_1, y_2, \ldots, y_m)$ for $0 < \varepsilon < y_i'' - y_i'"

examples: ($s : \mathbb{R} \rightarrow \mathbb{R}$, strictly convex, strictly monotonic)

\[ g(y) = \Sigma s(y_i) \]

so, e.g. $g(y) = \Sigma y_i^k$, $k > 1$
Classic fairness criteria give only one “Best” solution

- $\sum f_i$
- $\min \max f_i$
- lexmin $\rightarrow$ generalizes min max; $\rightarrow$ OWA with infinite weight difference
- $\sum -\log f_i$ $\rightarrow$ ok, as strictly Schur convex
- Nash Bargaining Solution $\rightarrow$ not compatible
Sum of criteria is too weak to produce an equitable solution.

$$\min \left( \sum C^G_i + \sum C^R_i \right) = \min \sum C_i$$

Minimized by Shortest Processing Time (SPT) schedule:

This is unfair to red agent... Pairs of jobs have to be permuted

But should it be the only solution?

$$1 \quad 1 \quad 2 \quad 2 \quad \ldots \quad 2^p \quad 2^p$$

$$1 \quad 1 \quad 2 \quad 2 \quad \ldots \quad 2^p \quad 2^p$$

$$1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 2$$

$$1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 2$$
MIN MAX CAN RESULT IN A VERY INEFFICIENT SOLUTION (...BUT LEXMIN IS EQUITABLY OPTIMAL)
Nash Bargaining Solution takes into account best alternative to agreement.

\[ \min (g_1 - f_1) (g_2 - f_2) \]  

(Agnetis, 2008)

How to choose \( g \)? What is the best alternative?
All Equitable Solutions

Pareto-prune k best sum
In the worst case, there are exponentially many equitably-optimal solutions...

Red jobs delayed by $2^{p+1} - 1$
but last green jobs adds to green agents’ goal: $2(2^{p+1} - 1) + 2^{p+1}$
Thus, this schedule favours red by: $2^{p+2} - 3$

After switching order of two jobs ($i \leq p$):

- red: $+2^i$
- green: $-2^{i-1}$
- sum: $+2^{i-1}$

Generalized Lorentz Dominance: ($x_i = 1$ if there was a switch)
$[C_1 - \sum x_i 2^i ; C_2 + \sum x_i 2^i]$  
($C_1 < C_2$, both are constants)

Pareto-independent; thus all possible switches equitably-independent...
Pareto-prune: Find the complete Pareto-optimal set, then remove equitably dominated solutions.

Possible problem: too many Pareto-optimal solutions
Applied to multi-agent scheduling: dynamic programming for Pareto-opt schedules

Pareto-opt schedules with $n$ jobs:
- extend Pareto-opt schedules with $n-1$ jobs
- remove Pareto-dominated combinations

Order between jobs of the same owner: SPT
Thus schedule specifies owners of the first $n$ jobs
K-BEST SUM: DETERIORATING MIN SUM, FIND EQUITABLY-OPTIMAL SOLUTIONS

[Perny et al, 2006]
Finding equitable schedule for the same total score

- find chains of jobs of the same length
- starting from the chain of longest job, greedy reduce the difference of payoffs

Why it works?

- after each chain, the difference of payoffs is decreased
- by starting from the longest jobs, we are able to reduce “errors”
Finding next degradation

- in current schedule, for each pair of jobs: (longer after shorter)

- switch such pair

- compute total score: $\sum C_i += (m-l)$

- add to not visited schedules (heap by $\sum C_i$)

- take the smallest element from heap
Bounding the number of steps

\[ f_1(x) + f_2(x) = 2A \]

2 criteria minimization

[B, A]

A, B

no equitable solutions here

eq worse

equitably better

equitably equal

pareto better
Equitable multi-agent optimization: conclusions

- multi-agent optimization becomes important (shared system, centralized control)
- “true” multicriteria approach to fair optimization (many solutions to chose from)
- single equitable solution: generalization of some known concepts
- all equitable solutions: can be intractable... (→approximation algorithms?)
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Thank you!