From Altruism to Non-Cooperation in Routing Games

Amar P. Azad

Joint work with Eitan Altman, INRIA, MAESTRO Group, Sophia Antipolis
Rachid ElAzouzi, University of Avignon, LIA/CERI.

July 11, 2008

Outline

1. Model and Problem Formulation
   - Routing Game
   - Cooperation Paradigm
   - Problem Formulation
   - Network Topology with Cooperation
2. Numerical Investigation
   - Experiments
   - Observations Summary
3. Existence and Uniqueness of NEP
   - Assumptions
4. Summary

Outline

1 Model and Problem Formulation
- Routing Game
- Cooperation Paradigm
- Problem Formulation
- Network Topology with Cooperation

2 Numerical Investigation
- Experiments
- Observations Summary

3 Existence and Uniqueness of NEP
- Assumptions

4 Summary

Outline

1. Model and Problem Formulation
   - Routing Game
   - Cooperation Paradigm
   - Problem Formulation
   - Network Topology with Cooperation
2. Numerical Investigation
   - Experiments
   - Observations Summary
3. Existence and Uniqueness of NEP
   - Assumptions
4. Summary

Outline

1. Model and Problem Formulation
   - Routing Game
   - Cooperation Paradigm
   - Problem Formulation
   - Network Topology with Cooperation
2. Numerical Investigation
   - Experiments
   - Observations Summary
3. Existence and Uniqueness of NEP
   - Assumptions
4. Summary

Outline

1. Model and Problem Formulation
   - Routing Game
   - Cooperation Paradigm
   - Problem Formulation
   - Network Topology with Cooperation
2. Numerical Investigation
   - Experiments
   - Observations Summary
3. Existence and Uniqueness of NEP
   - Assumptions
4. Summary

System Model

- Network: a graph $\mathcal{G} = (\mathcal{V}, \mathcal{L})$
  - $\mathcal{V}$ is a set of nodes
  - $\mathcal{L} \subseteq \mathcal{V} \times \mathcal{V}$ is set of directed links.
- $\mathcal{I} = \{1, 2, ..., I\}$ is a set of users which share the network $\mathcal{G}$.
- f_l^i= flow of user i in link l .
- Each user i has a throughput demand rate r^i (which can be split among various path).
- Strategy: $\mathbf{f}^i = (f_l^i)_{l \in \mathcal{L}}$ is the routing strategy of user i.

Assumptions:

- At least one link exist between each pair of nodes(in each direction).
- Flow is preserved at all nodes.

Nash Equilibrium

- Cost/Utility function $J^i(\mathbf{f}) = \sum_l f_l^i \mathcal{T}_l(f_l)$.

Each user seeks to minimize the cost function J^i, which depends upon routing strategy of user i as well as on the routing strategy of other users.

Nash Equilibrium

A vector $\tilde{\mathbf{f}}^i$, $i = 1, 2, ..., I$ is called a Nash equilibrium if for each user i, $\tilde{\mathbf{f}}^i$ minimizes the cost function given that other users' routing decisions are $\tilde{\mathbf{f}}^j$, $j \neq i$. In other words,

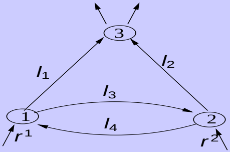
$$\tilde{J}^i(\tilde{\mathbf{f}}^1, \tilde{\mathbf{f}}^2, ..., \tilde{\mathbf{f}}^I) = \min_{\mathbf{f}^i \in \mathbf{F}^i} \hat{J}^i(\tilde{\mathbf{f}}^1, \tilde{\mathbf{f}}^2, ..., \mathbf{f}^i, ..., \tilde{\mathbf{f}}^I), \quad i = 1, 2, ..., I \quad , \tag{1}$$

where $\mathbf{F}^i$ is the routing strategy space of user i.

Network Topology

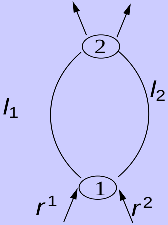
Consider the following network topology

Load Balancing Network



$$\hat{J}^i = \sum_{l \in \{1,\ldots 4\}} f_l^i T_l(f_l)$$

Parallel Link Network



$$\hat{J}^i = \sum_{l \in \{1,2\}} f_l^i T_l(f_l)$$

Cost Function

Consider the following Cost function.

Linear Cost Function

- Used in Transportation Networks
- $T_l(f_{l_i}) = a_i f_{l_i} + g_i$ for link $i = 1, 2$, where as, $T_l(f_{l_j}) = c f_{l_j} + d$ for link $j = 3, 4$.

M/M/1 Delay Cost Function

- Used in Queueing Networks
- $T_l(f_{l_i}) = \frac{1}{C_{l_i} - f_{l_i}}$, where the C_{l_i} and f_{l_i} denote the total capacity and total flow of the link l_i.

For parallel link topology only link l_i, $i = 1, 2$ exist while for load balancing topology link l_i, $i = 3, 4$ also exist.

Selfish Users

Some results for selfish users (with some assumptions)

- Orda et al has shown unique Nash equilibrium for Parallel link network with MM1 cost function.
- Kameda et al also claim unique Nash equilibrium for Load balancing network with MM1 cost function.
- Braess like paradox is observed by Kameda et al in Load balancing network with MM1 cost function.

Outline

1. Model and Problem Formulation
   - Routing Game
   - Cooperation Paradigm
   - Problem Formulation
   - Network Topology with Cooperation
2. Numerical Investigation
   - Experiments
   - Observations Summary
3. Existence and Uniqueness of NEP
   - Assumptions
4. Summary

What happens when there is some Cooperation ?

Degree of Cooperation

Definition

Let $\overrightarrow{\alpha^i} = (\alpha^i_1, .., \alpha^i_{|\mathcal{I}|})$ be the *degree of Cooperation* for user i. The new operating cost function $\hat{J}^i$ of user i with Degree of Cooperation, is a convex combination of the cost of user from set $\mathcal{I}$,

$$\hat{J}^i(\mathbf{f}) = \sum_{k \in \mathcal{I}} \alpha^i_k J^k(\mathbf{f}); \;\; \sum_k \alpha^i_k = 1, i = 1, ...|\mathcal{I}|$$

- Non cooperative user : $\alpha^i_i = 1 \Rightarrow$ User i takes into account of only its cost
- Cooperative (Equally cooperative) - $\alpha^i_j = \frac{1}{|\mathcal{P}|}$, where, $j \in \mathcal{P}, \mathcal{P} \subseteq \mathcal{I} \Rightarrow$ User i takes into account the cost of each users j(including itself).
- Beyond Cooperation - Altruistic user : $\alpha^i_i = 0 \Rightarrow$ User i takes into account the cost of only other users

Outline

1. Model and Problem Formulation
   - Routing Game
   - Cooperation Paradigm
   - Problem Formulation
   - Network Topology with Cooperation
2. Numerical Investigation
   - Experiments
   - Observations Summary
3. Existence and Uniqueness of NEP
   - Assumptions
4. Summary

With Cooperation

Each user still seeks to minimize the operating cost function $\hat{J}^i$.

Non-Cooperative Framework

We can benefit to apply the properties of non-cooperative games. e.g. (Nash Equilibrium etc.)

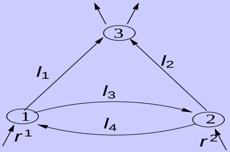
Outline

1. Model and Problem Formulation
   - Routing Game
   - Cooperation Paradigm
   - Problem Formulation
   - Network Topology with Cooperation
2. Numerical Investigation
   - Experiments
   - Observations Summary
3. Existence and Uniqueness of NEP
   - Assumptions
4. Summary

Network Topology

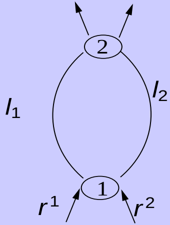
Consider the following network topology

Load Balancing Network



$$\hat{J}^i = \sum_{l \in \{1,\ldots 4\}} \sum_{k \in \{1,2\}} \alpha_k^i f_l^k T_l(f_l)$$

Parallel Link Network



$$\hat{J}^i = \sum_{l \in \{1,2\}} \sum_{k \in \{1,2\}} \alpha_k^i f_l^k T_l(f_l)$$

Related work

On Various degree of Cooperation

Michiardi Pietro, Molva Refik A game theoretical approach to evaluate cooperation enforcement mechanisms in mobile ad hoc networks WiOpt'03

On Altruism

Handbook of the Economics of Giving, Altruism and Reciprocity, Volume 1, 2006, Edited by Serge-Christophe Kolm and Jean Mercier Ythier

"Motivationally, altruism is the desire to enhance the welfare of others at a net welfare loss to oneself."

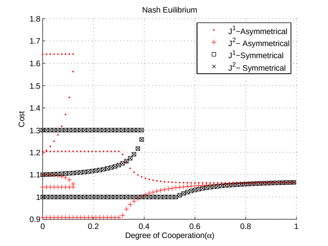
Outline

1. Model and Problem Formulation
   - Routing Game
   - Cooperation Paradigm
   - Problem Formulation
   - Network Topology with Cooperation
2. Numerical Investigation
   - Experiments
   - Observations Summary
3. Existence and Uniqueness of NEP
   - Assumptions
4. Summary

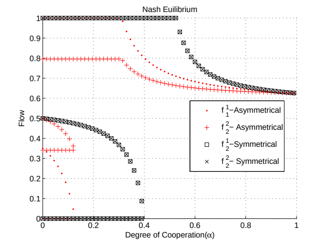
Load Balancing Network with Linear link Cost

Parameters : $a = 1, c = 0, d = 0.5$, Cooperation : $\{$ Symmetrical: $\alpha^1 = \alpha^2$, Asymmetrical: $0 \leq \alpha^1 \leq 1, \alpha^2 = 1\}$

Cost at Nash Equilibrium



Flow at Nash Equilibrium



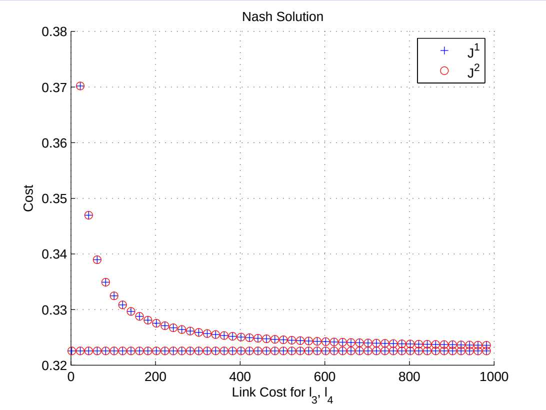
Some strange observation with Cooperation

- Multiple Nash equilibrium - Pure and Mixed Nash Equilibria...

Braess like Paradox

Parameters : $a_1 = a_2 = 4.1, d = 0.5$, Cooperation : { Symmetrical: $\alpha^1 = \alpha^2 = 0.07$, Asymmetrical: $0 \leq \alpha^1 \leq 1, \alpha^2 = 1$}

Cost at Nash Equilibrium

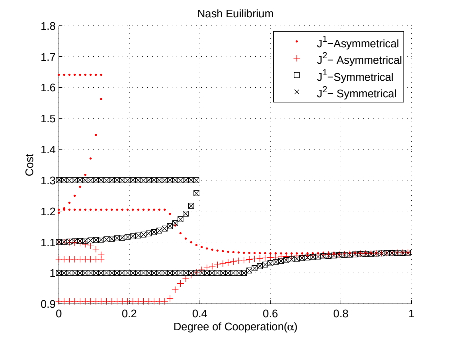


Braess like Paradox: Additional resources degrades the performance.

Even More - Paradox in Cooperation

Parameters : $a = 1, c = 0, d = 0.5$, Cooperation : { Symmetrical: $\alpha^1 = \alpha^2$, Asymmetrical: $0 \leq \alpha^1 \leq 1, \alpha^2 = 1$}

Cost at Nash Equilibrium

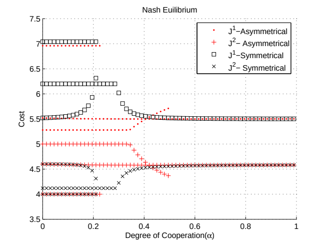


Paradox in Cooperation: Low Cooperation degrades (the cost) !..
Selfishness is not always good :)
Altruism behavior may help some time.

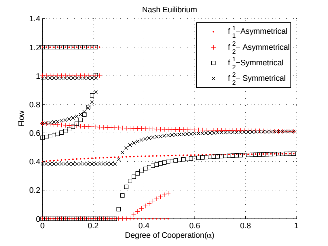
Parallel Link Network with Linear link Cost

Parameters : $a = 1, c = 0, d = 0.5$, Cooperation : { Symmetrical: $\alpha^1 = \alpha^2$, Asymmetrical: $0 \leq \alpha^1 \leq 1, \alpha^2 = 1$}

Cost at Nash Equilibrium



Flow at Nash Equilibrium

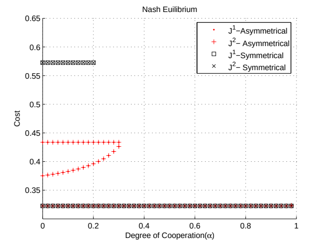


Similar Observations.

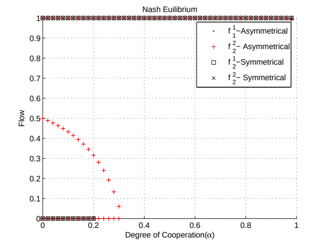
Load balancing network with M/M/1 link cost

Parameters : $a = 1, c = 0, d = 0.5$, Cooperation : { Symmetrical: $\alpha^1 = \alpha^2$, Asymmetrical: $0 \le \alpha^1 \le 1, \alpha^2 = 1$}

Cost at Nash Equilibrium



Flow at Nash Equilibrium

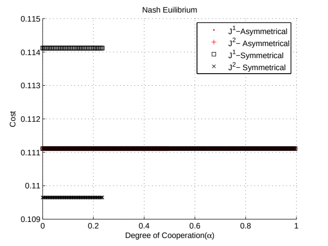


Multiple Nash Equilibria.

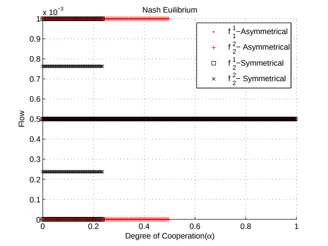
Parallel link with M/M/1 link cost

Parameters: Cl1 = 0:001;Cl2 = 0:001; r1 = 1; r2 = 1, Cooperation : $\{$ Symmetrical: $\alpha^1 = \alpha^2$, Asymmetrical: $0 \leq \alpha^1 \leq 1, \alpha^2 = 1\}$

Cost at Nash Equilibrium



Flow at Nash Equilibrium



Multiple Nash Equilibria.

Outline

1. Model and Problem Formulation
   - Routing Game
   - Cooperation Paradigm
   - Problem Formulation
   - Network Topology with Cooperation
2. Numerical Investigation
   - Experiments
   - Observations Summary
3. Existence and Uniqueness of NEP
   - Assumptions
4. Summary

Observation Summary

- Uniqueness of NEP is lost
- Paradox in Cooperation
- Braess like paradox

Outline

1 Model and Problem Formulation
- Routing Game
- Cooperation Paradigm
- Problem Formulation
- Network Topology with Cooperation

2 Numerical Investigation
- Experiments
- Observations Summary

3 Existence and Uniqueness of NEP
- Assumptions

4 Summary

Assumptions on Cost function

Orda et al has shown uniqueness for Nash equilibria in non-cooperative scneraio.. Following Orda et al, Consider the following assumption on the Cost function J^i

Type G function- Assumptions

G1: $J^i(\mathbf{f}) = \sum_{l \in \mathcal{L}} J^i_l(f_l))$. Each J^i_l satisfies:
G2: J^i_l:$[0, \infty) \rightarrow (0, \infty]$ is continuous function.
G3: J^i_l: is convex in f^j_l for $j = 1, ...|\mathcal{I}|$.
G4: Wherever finite, J^i_l is continuously differentiable in f^i_l, denote $K^i_l = \frac{\delta \hat{J}^i_l}{\delta f^i_l}$.

Assumptions on Cost function

Type B function- Assumptions

B1: $J^i(\mathbf{f}) = \sum_{l \in \mathcal{L}} f_l^i T_l(f_l))$

B2: $T_l : [0, \infty) \rightarrow (0, \infty]$.

B3: $T_l(f_l)$ is positive, strictly increasing and convex.

B4: $T_l(f_l)$ is continuously differentiable.

Type C function

C1: $\hat{J}^i(f_l^i, f_l) = f_l^i T_l(f_l)$ is a type-B cost function.

C2: $$T_l = \begin{cases} \frac{1}{C_l - f_l} & f_l < C_l \\ \infty & f_l > C_l \end{cases}.$$

Where C_l is the capacity of the link l.

Note that type C is a special kind of type B function which correspond to M/M/1 delay function. Orda et al has shown unique Nash solutions for type B functions.

Existence and Uniqueness of NEP with Cooperation

Cost functions

$$\hat{J}_l^i(\mathbf{f}) = (\alpha^i f_l^i + (1-\alpha^i) f_l^{-i}) T_l(f_l)$$
$$= ((2\alpha^i - 1) f_l^i + (1-\alpha^i) f_l) T_l(f_l)$$

Existence can be directly guaranteed by Orda et al.

Uniqueness of NEP

- for $\alpha^i \geq 0.5$ - Unique - Directly by Orda et al Using Kuhn Tucker condition
- for $\alpha^i < 0.5$ - Not Unique (Because $K_l^i(f_l^i, f_l)$ is strictly increasing function in f_l^i). and f_l.

Concluding Remarks

We parameterize the ”degree of Cooperation” to capture the behavior in the regime from altruistic to egocentric and identify some strange behavior

- Loss of uniqueness
- Cooperation paradox
- Braess Paradox

Ongoing direction

- Detailed mathematical study of uniqueness
- Characterization for more general network.

References

- Ariel Orda, Raphael Rom, and Nahum Shimkin, " Competitive Routing in Multiuser Communication Networks",*IEEE/ ACM Transactions on Networking*, Vol.1 No. 5, October 1993
- Y. A. Korilis, A. A. Lazar and A. Orda, "Architecting Non cooperative Networks", *IEEE Journal on Selected Areas in Communications* N. 13(7), pp. 1241–1251, 1995.
- H. Kameda , E. Altman, T. Kozawa, Y. Hosokawa , "Braess-like Paradoxes in Distributed Computer Systems" , *IEEE Transaction on Automatic control*, Vol 45, No 9, pp. 1687-1691, 2000.
- Pietro Michiardi, Refik Molva, "Analysis of coalition formation and cooperation strategies in mobile adhoc netowrks", *Ad Hoc Networks* , Volume 3 N$^{\circ}$ 2, March 2005 , pp 193-219
- T. Jimenez, E. Altman, T. Basar and N. Shimkin, "Competitive routing in networks with polynomial costs" *IEEE Trans. on Automatic Control 47*, Jan. 2002, pp. 92-96

Thanks