PSI2 : Envelope Perfect Sampling of Non Monotone Systems

Ana Bušić¹, Bruno Gaujal²,³
Gaël Gorgo²,³ and Jean-Marc Vincent²,³

¹INRIA/ENS Paris ²INRIA-Rhône-Alpes ³Laboratoire d’Informatique de Grenoble

Outline

1 Motivations
2 Perfect Sampling
3 Sampling efficiency
4 Future Work and References

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### Motivations

#### Perfect Sampling of Complex Large Scale Markov Chains

#### Applications
- Finite queuing networks (dynamic routing)
- Call centers
- Grid/cluster scheduling
- Rare event estimation
- Statistical verification of program models

#### Models
- Discrete vector state-space $\mathcal{X}$
- Event based models
  \[ X_{n+1} = \Phi(X_n, e_{n+1}), \quad e_n \in \mathcal{E} \]
  Stochastic recurrence equation
- Independent events (iid)

Provide **independent** samples of **stationary** states.

#### PSI2: a Perfect Sampler
- Library of **monotone** events
- Simulation kernel
- Efficient simulator: polynomial in the model dimension

⇒ **Extension to non-monotone events**
Perfect Sampling Principle

Synchronizing pattern $\Rightarrow$ finite backward scheme $\tau^* < \infty$

[NSMC 2003, LAA 2004]
Perfect Sampling

Monotone Perfect Sampling

\[
X \to (n + 1) \to X
\]

same convergence condition

complexity in \( O(\mathbb{E}T^*) \) \( \Rightarrow \) polynomial in model dimension

[NSMC 2006, QEST 2008]
Synchronizing pattern for envelopes

**complexity unknown** but practically efficient
Envelopes and Splitting Perfect Sampling

Guarantees the convergence
complexity unknown but practically more efficient

[VALUETOOLS 2008]
Sampling efficiency

Batch arrivals

Batch arrival $B$

$\lambda \quad C \quad \mu \quad C \quad \mu \quad C \quad \mu \quad C \quad \mu$

$C = 200 \quad \mu = 1 \quad B = 2$ with probability $\frac{1}{2}$ and $B = 3$ with probability $\frac{1}{2}$

$\Rightarrow$ Almost monotone systems
Sampling efficiency

Coxian queues

\[ \lambda = \frac{C}{\mu} \]

\[ C = 1000 \quad p = \frac{1}{2} \quad \mu_2 = 2 \quad \lambda = 1 \]

\[ \Rightarrow \text{pre-computation for phase-type distribution} \]
Future Work and References

Synthesis

Non-monotone events

- Negative customers, join, ...(frontier in 0)
- Batch arrivals or services (large coupling time ⇒ splitting)
- Event triggering on non-monotone condition
- Coxian and phase-type distribution (transform the state-space)
- ...

ψ² Implementation

- adaptation of the kernel
- user defined envelopes process
- basic events in the library

Some references

- Methodological reference

- Non-monotone load-sharing policies
  G. Gorgo, J-M Vincent. Perfect Sampling of Load Sharing Policies in Large Scale Distributed Systems. ASMTA, LNCS, 6148

- Performance comparison in statistical model-checking
Download: http://gforge.inria.fr/projects/psi