Bounding the Performance of BCMP Networks with Load-Dependent Stations

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Abstract

In this paper, we introduce new bounds on the system throughput and response time of closed, single-class BCMP queueing networks with load-dependent stations. Under the assumption that stations relative service rates are non-decreasing functions of their queue lengths, the bounds derive from the monotonicity of system throughput and queue-lengths and exploit the asymptotic equivalence that exists between closed and open single-class BCMP networks when the number of jobs N populating a closed network grows to infinity. The bounds can be applied when N is sufficiently large and the minimum N which allows their use is given. Experimental results present scenarios in which the proposed bounds significantly improve the accuracy of existing techniques and we analytically show that they are always more accurate than the popular balanced job bounds when N is greater than a given threshold. [1]

I. Introduction

Closed, single-class, BCMP queueing networks models [3] are a powerful, robust tool which in the last decades has been widely adopted to analytically evaluate the performance of computer and communication systems (see, e.g., [5], [13], [31] for recent works). Their effectiveness is due to the good compromise they provide between the accuracy and the computational effort needed by the model solution. The continuous, on-line solution of such queueing network models is often required in the management of modern data centers which, to achieve the best performance, iteratively solve several complex models. In fact, modern computer systems are highly dynamic, self-optimizing, self-configuring [4], [9], [1], and in such frameworks, the efficient solution of several queueing network models is required whenever the system *changes* in order to find the best *system configuration* which optimizes performance indices and guarantees given quality of service constraints. The fulfillment of quality constraints and the optimization of performance indices can be approached through the formulation of non-linear optimization problems [1], [2], [5] which make a large use of performance indices in their objective functions and constraints. In such optimization problems, an *on-line* evaluation of performance indices significantly reduces the computational requirements. Unfortunately, no exact on-line solution algorithm is known for the whole class of closed, single-class BCMP queueing networks.

The bounding analysis of closed queueing networks can support the above requirements because it is aimed to provide computationally simple formulas which bound performance indices giving the qualitative behavior of the system. Given that in general it is difficult to analytically evaluate which bounds are the tightest ones, the efficiency of this analysis is such that a number of bounding techniques can be simultaneously executed when evaluating the qualitative behavior of a system and, a posteriori, the tightest one can be chosen with a small computational effort. Clearly, if an upper (lower) bound is needed, then the lowest (highest) bound computed provides the best accuracy.

Many works have been developed in the literature supporting the bounding analysis of single-class, closed BCMP networks with *Load-Independent* (LI), or fixed-rate, stations [12], [32], [16], [10], [17], [29], [30], [14], [8], [7] but little appeared about networks with *Load-Dependent* (LD) stations. This type of station is useful to model a service center where its processing speed depends on its queue length and it is used in many applications. For instance, LD stations can represent multiple-server queues or flow-equivalent stations [6] which are used for the hierarchical modeling of large/multitiered networks. LD stations are also used to model, with a single station, the behavior of a sub-

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system which is difficult to treat analytically. In this case, the parameterization of the processing speeds is approached through measurements or simulations. Unfortunately, the exact analysis of BCMP networks with LD stations becomes more difficult and the temporal computational requirement is $O(MN^2)$ where N is the number of jobs populating the network and M is the number of service stations [6] (recall that for networks with LI stations only this requirement drops to $O(MN)$). Moreover, current solution algorithms exhibit numerical instabilities that strongly affect the accuracy of the solution and eventually yield unfeasible solutions, e.g. negative throughputs [23]. We emphasize that even single-class networks can give rise to numerical instabilities. A bounding analysis is aimed to overcome these drawbacks.

In [27], the authors express bounds of a network with LD stations in terms of bounds of a network with only LI stations in order to apply, in turn, the existing literature referenced above. However, we observe that this new (LI) network can be characterized by a different *bottleneck*, i.e. a different station having the highest mean *service demand*, and in this situation the resulting bounds can be drastically loose (it is well-known that the stations which strongly affect the performance of a queueing network are the bottlenecks). An illustrative example of this *bottleneck shifting* phenomenon is shown in the paper. Further, the same authors provide an additional bound under the assumption that the stations service speeds are increasing functions of the number of jobs in their queues. In this case, the network is again bounded by mean of a network with only LI stations but N is suitably scaled. This bound avoids the bottleneck shifting phenomenon. However, the population size scaling yields very loose bounds in many practical cases and significantly restricts the bound applicability. In [27], it is also shown that such bounds can be tightened if only multiple-server stations are considered. Bounds related to the special case of multiple-server stations are also proposed in [28], [22], [11].

In this paper, we propose new bounds on the system throughput and response time of single-class, closed BCMP networks with LD stations. We assume that the relative rate of service of each station is a non-decreasing function of the number of jobs in its queue. The bounds mainly follow from the monotonicity of system throughput [26] and queue-lengths [25] and exploit the asymptotic equivalence that exists between closed and open single-class BCMP networks when the number of jobs populating a closed network grows to infinity (see, e.g., [18]). The bounds can be applied when the number of jobs in the network is greater than or equal to a given threshold which depends on the parameters characterizing *non*-bottleneck stations. The bounds overcome the bottleneck shifting phenomenon mentioned above which can appear by applying ([27], Theorem 3.1) and alleviate the effects of the population size scaling of ([27], Theorem 3.2). With respect to well-known scenarios published in the literature, experimental results show that the proposed bounds provide improved accuracy in many cases. When N is sufficiently large, we prove that the throughput bound always provides better accuracy than the popular Balanced Job Bounds (BJB) [32], [17].

The structure of the paper is as follows. In Section II we introduce the model under investigation and the notation used in the paper. In Section III we derive the bounds discussing their analytical properties and comparing their accuracy with respect to the well-known class of BJB bounds. In Section IV we present some numerical results and, finally, Section V draws the conclusions of our work.

II. Model and Notation

We consider single-class, closed BCMP queueing networks models [3]. There are M stations and the number of jobs circulating in the network is N. If not otherwise specified, index i will range from 1 to M indexing network stations.

We denote by D_i the mean *service demand* (also known as relative utilization [6]) at station i and we recall that for a closed network it can be interpreted as the total mean time spent by a job at station i when using the network alone and visiting a reference station exactly once.

Let $x_i : \mathbb{N} \to \mathbb{R}^+$ be an arbitrary non-decreasing function. $x_i(n)$ represents the load-dependent rate of service of i when there are $n \leq N$ jobs in i *relative* to the service rate when $n = 1$, i.e. $x_i(1) = 1$. For instance, if in station i there are $k_i < \infty$ *identical* servers working in parallel, then the relative service rate is defined as

$$x_i(n) = \begin{cases} n, & \forall n \leq k_i \\ k_i, & \forall n > k_i. \end{cases} \tag{1}$$

Note that if $k_i = 1$ then station i is a Load-Independent (LI) station. For simplicity of notation, we define

$$D_i^*(n) \equiv D_i / x_i(n) \tag{2}$$

as the *effective* mean service demand required by a job to station i when n jobs are contained in i, and

$$D_i^* \equiv \lim_{n \to \infty} D_i / x_i(n). \tag{3}$$

We note that the existence of previous limit follows by the monotonicity of $x_i(n)$. We also denote by $n_i \leq$

N the minimum positive integer such that $D_i(n_i) = D_i(n) = D_i^*,\ n = n_i, \ldots, N$. Let also

$$D^*_{max} \equiv \max_i D^*_i \tag{4}$$

and b_{max} be the cardinality of set

$$\{i : D^*_i = D^*_{max}\}\,, \tag{5}$$

i.e. the number of stations with, in the limit, maximum service demand. We note that the stations belonging to set (5) can be characterized by different load-dependencies.

In this paper, we assume that

$$\sum_{i:D^*_i=D^*_{max}} n_i < N + b_{max}. \tag{6}$$

This assumption will be used in Section III.

Throughout the paper, we adopt the following notation for output indices:

- $Q_i(N)$: mean *queue length* (number of jobs) of station i,
- $X(N)$: mean *system throughput*, i.e. the mean throughput of jobs expected at an arbitrary reference station (say k),
- $R(N) = \sum_i R_i(N)$: mean *system response time*, i.e. the mean time between two successive departures of a job performed at an arbitrary reference station (say k).

We also denote by Q_i the mean queue length of i when the number of jobs in the network grows to infinity, i.e. $Q_i \equiv \lim_{N\to\infty} Q_i(N)$. It is well-known that such limit exists. For simplicity, in the following we will refer to model inputs and outputs dropping the word *mean*.

III. The Bounds

In this section we derive bounds on the system throughput and response time of closed, single-class BCMP networks with LD stations.

A. Bounds Derivation

Given a closed, BCMP network, we make the following replacements

$$\begin{aligned} &N \leftarrow N + b_{max} - \sum_{i:D^*_i=D^*_{max}} n_i \\ &D_i(n) \leftarrow D^*_i,\ \forall i \text{ s.t. } D^*_i = D^*_{max},\ n = 1, \ldots, N. \end{aligned} \tag{7}$$

Applying (7) to the queueing networks under investigation, it is known (see [27], Theorem 3.2) that the system throughput (respectively, response time) of the resulting network bounds from below (above) the system throughput (response time) of the original network. Hence, to derive the bounds, we first apply transformations (7) to the input queueing network and assumption (6), weaker than the one of ([27], Theorem 3.2), ensures that the new population size is greater than zero.

The following inequality is a direct consequence of the monotonicity of $Q_i(N)$ [25]

$$\begin{aligned} \sum_{i:D^*_i<D^*_{max}} Q_i \geq & \sum_{i:D^*_i<D^*_{max}} Q_i(N) \\ = & \; N - \sum_{i:D^*_i=D^*_{max}} Q_i(N) \\ = & \; N - b_{max} Q_m(N) \end{aligned} \tag{8}$$

where index m is such that $D^*_m = D^*_{max}$ and we recall that Q_i is the queue length of station i when $N \to \infty$. The fact that all Q_i in (8) exist and are finite will be shown at the end of this section when we will focus on their computation. Applying the MVA recursion [24] and taking into account that $X(N)$ is monotonically increasing [26], we have

$$\begin{aligned} Q_m(N) \; & \leq X(N)D_m(1 + Q_m(N-1)) \\ & \leq X(N)D_m(1 + X(N)D_m(1 + Q_m(N-2))) \\ & \leq \cdots \\ & \leq \frac{1 - (X(N)D_m)^{N+1}}{1 - X(N)D_m} - 1. \end{aligned} \tag{9}$$

Substituting (9) in (8), we obtain

$$\hat{Q} \geq N - b_{max} \frac{1 - (X(N)D_m)^{N+1}}{1 - X(N)D_m} + b_{max} \tag{10}$$

where we define

$$\hat{Q} \equiv \sum_{i:D^*_i<D^*_{max}} Q_i. \tag{11}$$

After some basic algebraic manipulations, we obtain the following polynomial in $X(N)$

$$\begin{aligned} &N - \hat{Q} - X(N)D_m(b_{max} + N - \hat{Q}) + \\ &\qquad b_{max}(D_m X(N))^{N+1} \leq 0 \end{aligned} \tag{12}$$

whose solutions provide bounds on system throughput. The solution of (12) can be approached by exploiting standard root-finding techniques, e.g. Newton's method, with suitable starting conditions obviously ranging between 0 and $1/D_m$. However, in order to skip the iterations of root-finding methods and obtain a simple formula, we consider the following expression

$$\begin{aligned} &N - \hat{Q} - X(N)D_m(b_{max} + N - \hat{Q}) + \\ &\qquad b_{max}(D_m X'(N))^N D_m X(N) \leq 0 \end{aligned} \tag{13}$$

with $X'(N)$ denoting a generic lower bound on $X(N)$ which provides a computationally simple lower bound on system throughput. In particular, rearranging (13) the following inequality must hold

$$X(N) \geq X^-(N) \tag{14}$$

where

$$X^-(N) = \frac{N - \hat{Q}}{D_m(b_{max} + N - \hat{Q}) - b_{max}(D_m X'(N))^N D_m} \tag{15}$$

is thus a lower bound on system throughput if $N - \hat{Q} \geq 0$.

Noting that $0 \leq D_m X'(N) < 1$, as $N \to \infty$ we have that $X^-(N)$ approaches $1/D_m$ which implies that the throughput bound is asymptotically correct. The upper bound on system response time is trivially obtained by applying Little's law [20]. In particular,

Theorem 1: If $N - \hat{Q} \geq 0$, then the system response time of a closed, single-class BCMP network is bounded from above by

$$R^+(N) = \frac{D_m(b_{max} + N - \hat{Q}) - b_{max}(D_m X'(N))^N D_m}{(N - \hat{Q})/N}. \tag{16}$$

In order to compute bound (16), we need a formula for $\hat{Q}$. We now show that $\hat{Q}$ can be computed by exploiting algorithms for open BCMP networks which in general are extremely efficient.

Definition 1: Two queueing networks are called equivalent if the averages of their output performance indices are equal.

Suppose that, given a closed BCMP network, there exists exactly one station m such that $D^*_m > D^*_i$, $i \neq m$, i.e. $b_{max} = 1$. In this setting, it is well-known (see, e.g., [26], [18]) that a closed, single-class BCMP network with N jobs is equivalent to the open network which is obtained by removing station m and injecting the variable arrival rate $\lambda(n) = 1/D^*_m(n)$ if the number of jobs in the network is $N - n$, with $N - n > 0$, otherwise 0. We note that this equivalent open network can be alternatively seen as an open network with variable arrival rate $\lambda(n) = 1/D^*_m(n)$, $\forall n$, which drops an incoming job whenever the number of jobs in the network is N.

Since the steady-state probability of having N jobs in the open network is non-null, the equivalent open network is characterized by a non-null throughput of dropped jobs for each finite N. However, when N grows to infinity, the throughput of dropped jobs must approach zero because the open queueing network must be ergodic [18] and the queue length of station $i \neq m$ converges to a finite limit. Thus, in this case we have that a closed network, in the limit, is equivalent to the open network which is obtained by removing m and injecting the constant arrival rate

$$\lambda = 1/D^*_m \tag{17}$$

which represents the maximum throughput achievable by the closed network. Hence, the value of Q_i is computed by applying standard algorithms of open BCMP networks.

Taking into account (7), such equivalence holds even when $b_{max} > 1$. In fact, we note that all the b_{max} identical stations i with $D^*_i = D^*_{max}$ (obtained by means of (7)) can be aggregated into a single LD station (call it station m) with effective service demand [21]

$$D^*_m(n) = D_i \frac{n + b_{max} - 1}{n}. \tag{18}$$

B. Iterative Scheme

Formula (15) can be easily exploited to derive a finite difference equation scheme. Given that both $X'(N)$ and $X^-(N)$, in (15), are lower bounds on system throughput, the accuracy of $X^-(N)$ can be refined by iterating over $X^-(N)$ until a given precision threshold (say ϵ) between two successive bound refinements is reached. As starting condition, one can choose $X^-(N) = 0$. This analysis is summarized in Algorithm 1 where ϵ is a positive real number which breaks the loop when the bound refinement becomes negligible. The response time bound is then obtained by applying Little's law.

Algorithm 1 Iterative scheme for $X^-(N)$

```
X⁻ ← 0;
repeat
    X⁻prev ← X⁻;
    X⁻ ← the value of (15) with X′(N) = X⁻prev;
until |X⁻prev − X⁻|/X⁻prev ≤ ε
return X⁻;
```

Theorem 2: The sequence of lower bounds X^- computed in the loop of Algorithm 1 is monotonically increasing for each real $\epsilon \geq 0$.

Proof: Let $a = N - \hat{Q}$ and $b = D_m(b_{max} + N - \hat{Q})$. Let also X^-_t be the value of X^- computed at the t-th iteration of Algorithm 1. The theorem statement can be proved by induction on t. The base case is immediately true because

$$X^-_1 = \frac{a}{b} > 0 = X^-_0. \tag{19}$$

As induction hypothesis, assume that $X^-_t > X^-_{t-1}$, $t \geq 1$. Since, for each $t \geq 1$, $0 \leq D_m X^-_t \leq 1$, $b > b_{max}(D_m X^-_{t-1})^N D_m$ and $b_{max}(D_m X^-_{t-1})^N D_m < b_{max}(D_m X^-_t)^N D_m$, the following inequality which proves the induction step holds

$$X^-_{t+1} = \frac{a}{b - b_{max}(D_m X^-_t)^N D_m} > \frac{a}{b - b_{max}(D_m X^-_{t-1})^N D_m} = X^-_t. \quad (20)$$

■

Since each X^- computed in the loop of Algorithm 1 must be less than $1/D_m$, the following corollary is straightforward.

Corollary 1 (of Theorem 2) The sequence of lower bounds X^- computed in the loop of Algorithm 1 with $\epsilon = 0$ is convergent.
This also means that Algorithm 1 halts in a finite number of steps.

C. Computational Complexity

The term which mainly affects the computation of (15) is $\hat{Q} \equiv \sum_{i:D^*_i < D^*_{max}} Q_i$. Since $\hat{Q}$ represents the number of jobs of the open BCMP network discussed in Section A when $N \to \infty$, it can be computed by applying existing efficient algorithms for open BCMP networks [18]. In particular, since the queue length of each open network station requires, in the worst case, $O(N)$ steps [6], the computation of (15) is $O((M - b_{max})N)$. The temporal computational complexity of Algorithm 1 is, thus,

$$O((M - b_{max})N + \text{“\#inner iterations”}) \quad (21)$$

where “#inner iterations” denotes the number of iterations perfomed by the repeat-until loop. However, we notice that for many load-dependencies of practical interest there exist simple closed-form formulas which can significantly reduce this computational bound, e.g. constant/linear piece-wise LD stations, load-dependencies of delay stations (Infinite Servers), Heffes stations [15], multiple-server stations, limited queue-dependent stations [23], [18], etc. For instance, assuming the Heffes load-dependence for station i, i.e.

$$D^*_i(n) = D_i \frac{n + c_i}{n}, \ c_i > 0, \quad (22)$$

it is possible to show that

$$Q_i = (c_i + 1)\frac{\lambda D_i}{1 - \lambda D_i} \quad (23)$$

where λ is given by (17).

D. Comparison with BJB Bounds

We now compare the tightness of the proposed throughput bound with respect to the popular BJB lower bounds [32], [17]. It is known that these latter bounds refer to BCMP networks with LI and/or delay stations only. We recall that the BJB bound expression is given by [17]

$$X^-_{BJB}(N) = \frac{N}{Z + D + D_{max}(N-1) - D_{max}X'(N-1)Z} \quad (24)$$

where $D \equiv \sum_i D_i$, $X'(N-1)$ is a (generic) lower bound on $X(N)$ and Z is the sum of the service demands of all the delay stations. Since

$$X^-_{BJB}(N) \leq \frac{N}{D + D_{max}(N-1)}, \quad (25)$$

the proposed bound (15) is tighter than (24) if (sufficient condition)

$$\frac{N}{D + D_{max}(N-1)} < \frac{N - \hat{Q}}{D_{max}(N - \hat{Q} + b_{max})} \quad (26)$$

which holds true if and only if

$$N > \frac{\hat{Q}(D - D_{max})}{D - D_{max}(b_{max} - 1)}. \quad (27)$$

Since the right-hand term of (27) is positive and does not depend on N, this means that the proposed bound (15) eventually becomes always tighter than the BJB bound (24). Note that if $b_{max} = 2$, then (27) reduce to $N > \hat{Q}$ which implies that (15) provides improved accuracy whenever it can be applied.

IV. Experimental Results

In this section, we numerically compare the accuracy of the proposed bound (15) with respect to existing bounds in some scenarios. A preliminary accuracy evaluation of this bound has been already carried out *analytically* in the previous section.

As outlined in the introduction, the bounding schemes presented in [27] bound the throughput of a closed BCMP network with LD stations in term of bounds of a network with only LI stations. Thus, to obtain an *on-line* solution, the well-understood bounding analysis of LI stations can be applied. However, we note that these (bounded) networks with only LI stations can be characterized by

- a different bottleneck, i.e. a different station with highest effective service demand for large N, and

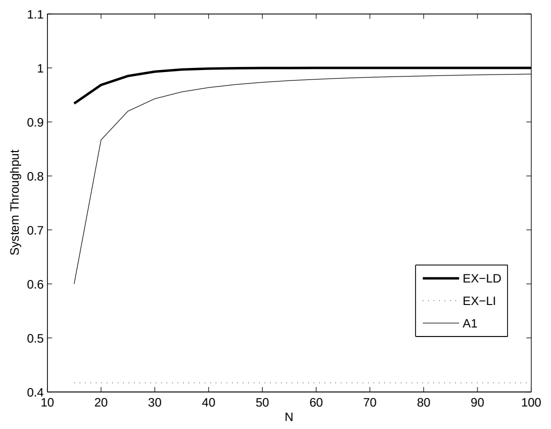

Fig. 1. The effect of the bottleneck shifting phenomenon on system throughput.

figure

- a very small population size.

Both issues can have dramatic effects on the accuracy of performance indices. In order to better understand the bottleneck shifting phenomenon and the population size scaling effects from a practical point of view, let us consider a simple three-station network with $D_1 = 1.0$, $D_2 = 0.8$, $D_3 = 0.6$ where station 2 is Heffes LD with $c_2 = 2$ and stations 1 and 3 are both LI. The bottleneck of this network is station 1. The related network with only LI stations considered by ([27], Theorem 3.1) is characterized by $D'_1 = 1.0$, $D'_2 = 2.4$, $D'_3 = 0.6$ and we note that in this new network the bottleneck is shifted to station 2. The related network with only LI stations considered by ([27], Theorem 3.2) is characterized by $D''_1 = 1.0$, $D''_2 = 0.8$, $D''_3 = 0.6$ and by the (scaled) population size $N = n_2 - 1 = 1$, and we note that this scaling is very loose (clearly, in this case we have $X(1) = 1/(D''_1 + D''_2 + D''_3) = 0.417$). For different N, Figure 1 illustrates the throughput bound computed with one iteration of Algorithm 1 (namely A1) and the exact system throughput of both the original LD network (EX-LD) and of the LI network (EX-LI). Clearly, the proposed bound (15) can be applied if

$$N \geq \hat{Q} = (1 + c_2)\frac{0.8}{1 - 0.8} + \frac{0.6}{1 - 0.6} = 13.5. \qquad (28)$$

It is evident that the bottleneck-shifting phenomenon yields a drastically loose bound. It is clear that for increasing values of c_2 (and, thus, D'_2) the accuracy of the resulting LI network which bounds the original one decreases. On the other hand, the proposed bounds are not affected by this drawback.

The scenario discussed above reveals a wide class of networks in which our bounds improve the approach [27]. Clearly, the fact that the proposed bound (15) is asymptotically correct (see Section III) implies that our approach is unaffected by the bottleneck shifting phenomenon. The fact that we derive bounds by scaling the population size only according to (7) implies that our approach alleviates the problem of the population size scaling which arises ([27], Theorem 3.2). Thus, this class of networks is composed of stations which satisfy the following constraints

- there exists a station i such that $D^*_i < D^*_{max} < \max_n D_i(n)$,
- there exist a subset of non-bottleneck stations such that the sum of all their n_i is large if compared to N.

We note that a pitfall of our approach is encountered when n_i is large with respect to N and i is such that $D^*_i = D^*_{max}$. In this case, the transformation (7) introduces a strong population scaling which can yield loose bounds. However, even the approach [27] is affected by this problem.

We now evaluate the accuracy of (15) in the well-know scenario presented in [17]. This scenario has been used to evaluate the accuracy of many existing bounding techniques for BCMP networks with LI stations only. In particular, we consider the following stress cases:

Stress case 1: The network is unbalanced with $M = 4$ and the service demands are $D_1 = D_2 = 0.1$, $D_3 = 0.05$ and $D_4 = 0.04$, i.e. $b_{max} = 2$.

Stress case 2: The network has the same stations of Stress Case 1 and an additional delay station (Infinite Server) with $D_5(n) = 1/n$ is introduced.

To evaluate the accuracy of our bounds, we introduce two load-dependencies in both stress cases above. In particular, we assume that stations 3 and 4 of all test cases are LD stations with effective service demands

$$D_i(n) = \begin{cases} 3D_i & n \leq 10 \\ 2D_i & 10 < n \leq 20 \\ D_i & 20 < n \end{cases}, \quad i = 3, 4. \qquad (29)$$

As an example, this type of load-dependency can model the behavior of a disk. In fact, when the number of requests inside the waiting buffer overflows given thresholds, the disk controller is able to optimize the reading/writing-heads movements or exploit cache data, and we assume that these optimizations reduce its service demands according to the piecewise function (29). On the other hand, if the queue length is small, i.e. less than 10, then it is difficult to perform such optimizations because the requests probably require the access to different disk sectors.

We note that for sufficiently large population sizes, the load-dependency (29) preserves, in term of system throughput, the qualitative behavior of both networks. Thus, the tests considered are good stress cases because, despite the introduction of (29), the network is still unbalanced [17] and because of the presence of multiple bottlenecks [19] which slow down the convergence speed of system throughput to its asymptotic value.

With respect to stress cases described above, Figures 2 and 3 compare the throughput bound computed with two iterations of Algorithm 1 (A2), the bound computed by applying ([27], Theorem 3.1) and ([27], Theorem 3.2) (respectively, S.Y.1 and S.Y.2), and the exact system throughput obtained by running the mean value analysis [23] (EX). The population size varies from 5 to 120 with step 5. The bounds obtained with three iterations of Algorithm 1 have not been shown because the accuracy improvement becomes negligible, namely less that 0.1%. In stress case 1 (respectively, 2), S.Y.1 and S.Y.2 have been obtained by applying the Geometric Bounds (Generalized Geometric Bounds) [8], [7] because they provide the best accuracy at the cost of a slightly higher computational effort than other bounding techniques for LI stations only. In Figures 2 and 3, we see that the proposed bound is closer to the exact throughput curve for the majority of the population sizes which allow its use. In contrast with S.Y.2, it provides improved accuracy even before the convergence of the (exact) throughput to its asymptotic value $1/D_1 = 10$. We also note that S.Y.1 provides the best accuracy for small population sizes, e.g. $N \leq 15$. However, for larger population sizes, the bottleneck shifting phenomenon makes S.Y.1 diverge to $1/(3D_3)$.

Now, let us consider again stress cases 1 and 2 with the following slight variation on the load-dependency definition (29)

$$D_i(n) = \begin{cases} 3D_i & n \leq 10 \\ 2D_i & 10 < n \leq 30 \\ D_i & 30 < n \end{cases}, \ i = 3, 4. \qquad (30)$$

Analogously, Figures 4 and 5 compare the throughput bound (15) with [27]. What we see is that our bound is remarkably closer than [27] to the throughput exact value (note the difference with the throughputs of Figures 2 and 3). This improvement is due to the fact that the new load-dependence (30) emphasizes, in the LI network obtained by applying ([27], Theorem 3.2), the population size scaling effect, which allows the use of S.Y.2 only when $N > 60$. It is clear that this pitfall can be emphasized again by changing the definition of (30) accordingly. Our bounds are unaffected by this problem. Even in this case, our bound provides improved accuracy before the convergence of the throughput to its asymptotic value.

The results shown in this section provide numerical evidence of the fact that our bound improves the accuracy of the approach [27] in many cases. However, we recall that its applicability requires that $N \geq \hat{Q}$. Since both bounding analyses are very efficient, we conclude that, when it is required an on-line performance evaluation of a system, one can simultaneously adopt both techniques and, a posteriori, choose the tightest one with little computational effort.

V. Conclusions

In this paper, we have proposed new bounds on the system throughput and response time of closed, single-class BCMP networks with load-dependent stations. Under the assumption of non-decreasing relative service rates, the bounds mainly derive from known analytical properties of the system throughput and the queue-lengths curves. The throughput bound has been analytically compared to the well-known balanced job bound and the analysis revealed that our method eventually becomes more accurate. We presented scenarios in which our bounds provide improved accuracy with respect to existing bounding techniques avoiding the bottleneck shifting phenomenon and alleviating the problem of the population size scaling.

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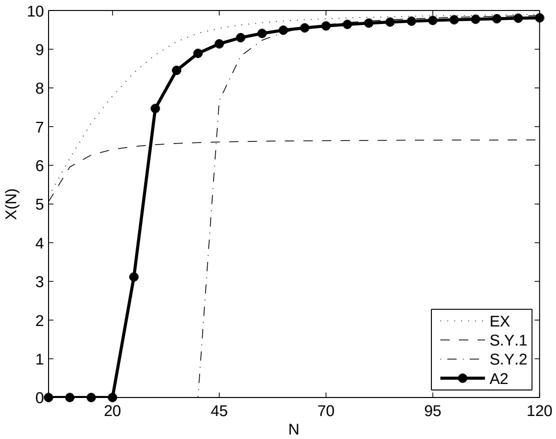

Fig. 2. Comparison on stress case 1 with the load-dependency (29).

figure

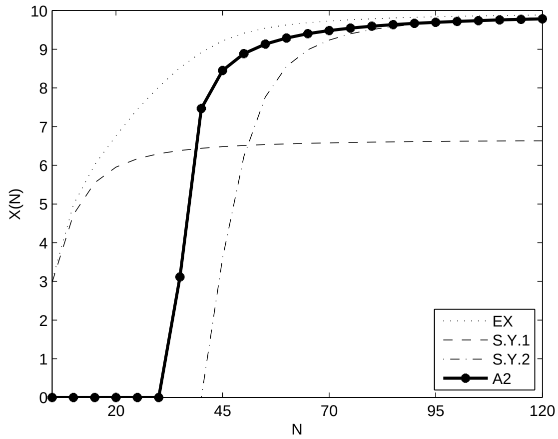

Fig. 3. Comparison on stress case 2 with the load-dependency (29).

figure

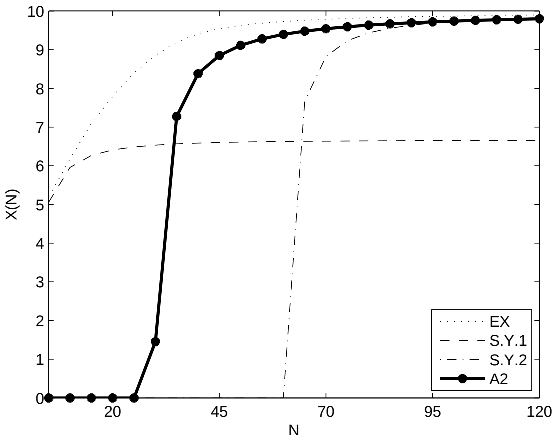

Fig. 4. Comparison on stress case 1 with the load-dependency (30).

figure

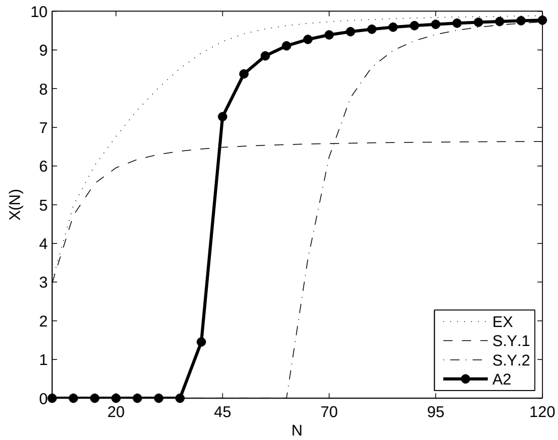

Fig. 5. Comparison on stress case 2 with the load-dependency (30).

figure

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