Impact of Demand-Response on the Efficiency and Prices in Real-Time Electricity Markets

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1Joint work with Jean-Yves Le Boudec (EPFL), Alexandre Proutiere (KTH) and Dan-Cristian Tomozei (EPFL)
Quiz: what is the value of energy?

Average price is 20$/MWh.
Average production is 0.

1. 0$.
2. 150k$
3. −150k$.

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   YES: If you are a private consumer.

2. $150k$

3. $-150k$

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   YES: If you buy on the real-time electricity market (Texas, mar 3 2012)

3. $−150$k$. 

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   YES: If you buy on the real-time electricity market (Texas, mar 3 2012)

3. −150k$.
   NO (but YES for the red curve! Texas, march 3rd 2012)
Can we understand real-time electricity prices?


Prices in 
$/MWh

Time of the day

Is it price manipulation or an efficient market?
Motivation and (quick) related work

Control by prices and distributed optimization

- *PowerMatcher: multiagent control in the electricity infrastructure* – Kok et al. (2005)
- *Real-time dynamic multilevel optimization for demand-side load management* – Ha et al. (2007)
- *Theoretical and Practical Foundations of Large-Scale Agent-Based Micro-Storage in the Smart Grid* – Vytelingum et al. (2011)

Fluctuations of prices in real-time electrical markets

- *Dynamic competitive equilibria in electricity markets* – Wang et al. (2012)
Issue: The electric grid is a large, complex system

It is governed by a mix of economics (efficiency) and regulation (safety).
Our contribution

We study a simple real-time market model that includes demand-response.

- Real-time prices can be used for control
  - Socially optimal
  - Provable and decentralized methods

- However:
  - There is a high price fluctuation
  - Demand-response makes forecast more difficult
  - Market structure provide no incentive to install large demand-response capacity
Outline

1. Real-Time Market Model and Market Efficiency
2. Numerical Computation and Distributed Optimization
3. Consequences of the (In)Efficiency of the Pricing Scheme
4. Summary and Conclusion
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We consider the simplest model that takes the dynamical constraints into account (extension of Wang et al. 2012)

Each player has internal utility/constraints and exchange energy
Two examples of internal utility functions and constraints

- Generator: generates $G(t)$ units of energy at time $t$.
  - Cost of generation: $cG(t)$.
  - Ramping constraints: $\zeta^- \leq G(t + 1) - G(t) \leq \zeta^+$. 
Two examples of internal utility functions and constraints

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- **Flexible loads**: population of $N$ thermostatic appliances: **Markov model**

  Consumption can be anticipated/delayed but
  - Fatigue effect
  - Mini-cycle avoidance

- **Internal cost**: temperature deadband.
- **Constraints**: Markov evolution and temperature deadband, switch on/off.
We assume perfect competition between 2, 3 or 4 players
(supplier, demand, storage operator, flexible demand aggregator)

Player $i$ maximizes:

$$\arg \max_{E_i \in \text{internal constraints of } i} \mathbb{E} \left[ \int_0^\infty \underbrace{W_i(t)}_{\text{internal utility}} - \underbrace{P(t)E_i(t)}_{\text{(spot price) \times (bought/sold energy)}} \, dt \right]$$
We assume perfect competition between 2, 3 or 4 players (supplier, demand, storage operator, flexible demand aggregator)

Player $i$ maximizes:

$$\arg \max_{E_i \in \text{internal constraints of } i} \mathbb{E} \left[ \int_{0}^{\infty} W_i(t) - P(t) E_i(t) \ dt \right]$$

Players share a common probabilistic forecast model

Players cannot influence $P(t)$. 

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Definition: a competitive equilibrium is a price for which players selfishly agree on what should be bought and sold.

\((P^e, E^e_1, \ldots, E^e_j)\) is a competitive equilibrium if:

- For any player \(i\), \(E^e_i\) is a selfish best response to \(P\):

\[
\arg \max_{E_i \in \text{internal constraints of } i} E \left[ \int_0^\infty W_i(t) - P(t)E_i(t) \, dt \right]
\]

- The energy balance condition: for all \(t\):

\[
\sum_{i \in \text{players}} E^e_i(t) = 0.
\]
An (hypothetical) social planner’s problem wants to maximize the sum of the welfare.

\[(E_1^e, \ldots, E_j^e) \text{ is socially optimal if it maximizes } \int_0^\infty \sum_{i \in \text{players}} W_i(t) \, dt,\]

subject to

- For any player \(i\), \(E_i^e\) satisfies the constraints of player \(i\).
- The energy balance condition: for all \(t\):

\[\sum_{i \in \text{players}} E_i^e(t) = 0.\]
The market is efficient (first welfare theorem)

**Theorem**

*For any installed quantity of demand-response or storage, any competitive equilibrium is socially optimal.*

If players agree on what should be bought or sold, then it corresponds to a socially optimal allocation.
Proof. The first welfare theorem is a Lagrangian decomposition

For any price process $P$:

\[
\max_{E_i \text{ satisfies constraints } i} \mathbb{E} \left[ \sum_{i \in \text{players}} \int W_i(t) dt \right] \geq \sum_{i \in \text{players}} \max_{E_i \text{ satisfies constraints } i} \mathbb{E} \left[ \int (W_i(t) + P(t)E_i(t)) dt \right]
\]

If the selfish responses are such that $\sum_i E_i(t) = 0$, the inequality is an equality.
Proof. The first welfare theorem is a Lagrangian decomposition

For any price process $P$:

$$\max_{E_i \text{ satisfies constraints } i} \mathbb{E} \left[ \sum_{i \in \text{players}} \int W_i(t) dt \right]$$

$$= \sum_{i \in \text{players}} \max_{E_i \text{ satisfies constraints } i} \mathbb{E} \left[ \int (W_i(t) + P(t)E_i(t)) dt \right]$$

If the selfish responses are such that $\sum_i E_i(t) = 0$, the inequality is an equality.
What is the price equilibrium? Is it smooth?
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- Production has ramping constraints,
- Demand does not.
Fact 1. Without storage or DR, prices are never equal to the marginal production cost (Wang et al. 2012)
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Fact 2. Perfect storage leads to a price concentration

Small storage

Large storage
Fact 3. Because of (in)efficiency, the price oscillates, even for large storage.

**Perfect storage**: price becomes equal to the marginal production cost.

**Realistic storage**: two modes in $\sqrt{\eta}$ and $1/\sqrt{\eta}$. 

Distribution has two modes.
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Reminder: If there exists a price such that selfish decisions leads to energy balance, then these decisions are optimal.

For any installed quantity of demand-response or storage:

- There exists such a price.
- We can compute it (convergence guarantee).
We design a decentralized optimization algorithm based on an iterative scheme.

**Iterative algorithm based on ADMM**

1. forecast price $P(1), \ldots, P(T), \bar{E}$
2. forecasts consumption $E$
3. Update price

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**Theorem**

*The algorithm converges.*
We use ADMM iterations.

Augmented Lagrangian:

\[ L_\rho(E, P) := \sum_{i \in \text{players}} W_i(E_i) + \sum_t P(t) \left( \sum_i E_i(t) \right) - \frac{\rho}{2} \sum_{t,i} \left( E_i(t) - \bar{E}_i(t) \right)^2 \]

ADMM (alternating direction method of multipliers):

\[ E^{k+1} \in \arg \max_E L_\rho(E, \bar{E}^k, P^k) \quad \text{for each player (distributed)} \]

\[ \bar{E}^{k+1} \in \arg \max_{\bar{E}} L_\rho(E^{k+1}, \bar{E}, P^k) \quad \text{projection (easy)} \]

\[ P^{k+1} := P^k - \rho \left( \sum_i E_i^{k+1} \right) \quad \text{price update} \]
ADMM converges because the problem is convex

1. Utility functions and constraints are convex
   - e.g., Ramping constraints, batteries capacities, flexible appliances
ADMM converges because the problem is convex

1. Utility functions and constraints are convex
2. We represent forecast errors by multiple trajectories

- Using covariance of data from the UK
ADMM converges because the problem is convex

1. Utility functions and constraints are convex
2. We represent forecast errors by multiple trajectories
3. We approximate the behavior of the flexible appliances by a mean-field approximation

Original system

Mean-field approximation
(limit as number of appliances is large)
The algorithm is distributed: each flexible appliance computes its best-response to price.

Object = Markov chain

\[
\begin{array}{c}
\text{undesirable states} \\
y=0 \\
y=Y_{\text{max}} \\
\text{no possible action when } y>0 \\
\text{possible action} \\
\end{array}
\]

\[
\begin{array}{c}
\text{on} \\
x=0 \\
x=X_{\text{max}} \\
\text{internal state} \\
\end{array}
\]

\[
\downarrow \text{best response}
\]

Sample trajectories of 5 fridges
Average x-state (mean field approx.)
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**Reminder:** we know how to compute a price such that selfish decision leads to a social optimum.

We can evaluate the effect of more flexible load / more storage.

- Is the price smooth?
- Impact on social welfare.
In a perfect world, the benefit of demand-response is similar to perfect storage.

No charge/discharge inefficiencies for demand-response (we can only anticipate or delay consumption).

$^{2}$The forecast errors correspond to a total wind capacity of 26GW.
Problem of demand-response: synchronization might lead to forecast errors

No Demand-response

Total consumption

Actual consumption is close to forecast
Problem of demand-response: synchronization might lead to forecast errors

No Demand-response
- Actual consumption is close to forecast

With Demand-response
- Problem if we cannot observe the initial state
Problem of demand-response. Non-observability is detrimental if the penetration is large

We assume that:

- The demand-response operator knows the state of its fridges
- The day-ahead forecast does not.

Social Welfare

\[ \text{Installed flexible power (in } \text{GW}^3) \]

\[^3\text{The forecast errors correspond to a total wind capacity of 26GW.}\]
Problem of the **market structure**. Incentive to install less demand-response than the social optimal.

Welfare for storage owner / demand-response operator

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\( ^4 \)The forecast errors correspond to a total wind capacity of 26GW.
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Summary

1. Real-time market model (generation dynamics, flexible loads, storage)

   \[ P(t) \]

   - Demand
   - Supplier
   - Flexible loads
   - Storage (e.g. battery)

2. A price such that selfish decisions are feasible leads to a social optimum.

3. We know how to compute the price.
   - Trajectorial forecast, mean field and ADMM

   Drawbacks: non-observability, under-investment
Perspectives

- Distributed optimization in smart-grid
  - In distribution networks.
  - Methodology:
    - Distributed Lagrangian (ADMM) is powerful
    - Use of trajectorial forecast makes it computable

- Optimization in Systems with many small agents.
- Virtual prices and/or virtual markets:
  - Bike-sharing systems (to solve the optimization problem but not to define prices for users).
Model and Forecast


Storage and Demand-response


ADMM


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