No Regrets: Distributed Power Control under Time-Varying Channels and QoS Requirements

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***Abstract*—The problem of power control in wireless networks consists of adjusting transmit power in order to achieve a target SINR level in the presence of noise and interference from other users. In this paper, we examine the performance of the seminal Foschini–Miljanic (FM) power control scheme in networks where channel conditions and user quality of service (QoS) requirements vary arbitrarily with time (e.g. due to user mobility, fading, etc.). Contrary to the case of static and/or ergodic channels, the system's optimum power configuration may evolve over time in an unpredictable fashion, so users must adapt to changes in the wireless medium (or their own requirements) "on the fly", without being able to anticipate the system's evolution. To account for these considerations, we provide a formulation of power control as an online optimization problem and we show that the FM dynamics lead to *no regret* in this dynamic context. Specifically, in the absence of maximum transmit power constraints, we show that the FM power control scheme performs at least as well as (and typically outperforms) any fixed transmit profile, irrespective of how the system varies with time; finally, to account for maximum power constraints that occur in practice, we introduce an adjusted version of the FM algorithm which retains the convergence and no-regret properties of the original algorithm in this constrained setting.**

I. Introduction

Ever since the early development stages of legacy wireless networks, power control has had a pivotal impact on wireless system design and operation. In particular, the rise of mobile telephony in the early 1990's cemented the need for distributed power control while the pioneering work of Zander [1, 2], Grandhi et al. [3, 4], Foschini and Miljanic [5] and Yates [6] paved the way for the introduction of efficient power control algorithms (both closed- and open-loop) in third generation CDMA-based cellular networks. Likewise, substantial effort has been made to optimize the performance of future and emerging network paradigms (such as ad hoc networks [7]) by analyzing connectivity and transport capacity under power modulation techniques whose goal is to minimize transmission power subject to user-specific quality of service (QoS) or quality of experience (QoE) requirements [8].

In a nutshell, power control allows wireless links to achieve their required throughput while minimizing the induced interference and the cost of energy consumption (both individually and at the network level). In this way, power control increases spatial spectrum reuse and, as a result, the capacity of the network and the battery life of wireless mobile devices deployed in the network. Thus, starting with seminal work by Foschini and Miljanic [5], several algorithms have been developed that provably allow receivers to meet signal to interference-plus-noise ratio (SINR) requirements of the form $\mathsf{SINR} \geq \gamma^*$ [5, 9–11]. However, while the benefits of power control algorithms are easy to evaluate in networks where channel conditions are static (or follow some stationary ergodic process), it is much harder to analyze their behavior in networks that evolve over time in an arbitrary, non-stationary fashion – e.g. due to users coming and going in the system or due to their QoS requirements changing as they switch between applications).

In more detail, the vast majority of works on power control examine the problem as a static optimization program or game, depending on the users' interactions and objectives – for a comprehensive survey, see e.g. [12, 13]. In particular, the standard setup for the feasibility and convergence analysis of power control is the static regime where transient phenomena are cast aside and both the channel conditions (channel gain and noise) and the QoS/QoE requirements of the involved connections are assumed effectively static – in the sense that they evolve slow enough for power control algorithms to converge. By contrast, in [10, 11], Holliday et al. explored the behavior of the well-known Foschini–Miljanic (FM) distributed power control algorithm [5] in the case of channels that evolve following a stationary ergodic process and provided sufficient conditions for the FM algorithm to converge in distribution.

In this paper, we drop such stationarity assumptions and we focus on systems that evolve *arbitrarily* over time in terms of both channel conditions and user QoS/QoE requirements. By so doing, standard approaches based on linear programming (for static channels) or stochastic convex optimization (for ergodically evolving channels), no longer apply because there is no underlying power problem to solve – either static or in the mean. Instead, our aim is to treat power control as a dynamically evolving optimization problem, and to employ techniques and ideas from *online* optimization in order to quantify how well the system's users adapt to changes in the wireless medium (and/or track their individually optimum transmit powers as they change over time).

In this dynamic framework, the most widely used performance criterion is that of *regret minimization*, a concept which was first introduced by Hannan [14] and which has since given rise to a vigorous literature at the interface of optimization, statistics, game theory, and machine learning –

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see e.g. [15, 16] for a comprehensive survey. Specifically, in the language of game theory, the notion of regret compares an agent's cumulative payoff over time to what he would have obtained by constantly playing the same action. Accordingly, regret minimization in our context corresponds to dynamic power control policies that perform asymptotically as well as the *optimum* fixed transmission power policy in hindsight, and *irrespective* of how the user's environment and/or requirements evolve over time.

In the context of power control, regret minimization has been used by [17] to study the transient phase of the FM algorithm in static environments, and to propose alternative convergent power control schemes that rely on the minimization of swap regret [18]. More recently, the authors of [19] proposed an online power control and allocation scheme with the aim of minimizing regret induced by low throughput in a cognitive radio (CR) network. By contrast, our goal here is to investigate the regret minimization properties of the original FM algorithm as proposed in [5] (and implemented in current CDMA-based networks). In the absence of maximum power constraints, we show that the FM algorithm leads to no regret with respect to any fixed transmit power profile. However, given that transmit powers can (and do) diverge to infinity in unstable systems, this result does not provide a *uniform* bound on the users' regret (which would signify the feasibility of the underlying power control problem).[1] Accordingly, we also study the regret minimization properties of the FM power control dynamics under maximum power constraints: to account for such constraints, we introduce a slight adjustment term on the original FM algorithm that maintains its convergence properties in static settings and which guarantees the dynamics' no-regret property, no matter how the wireless medium evolves over time. Importantly, this property continues to hold even if the users' requirements change over time (or if users come and go in the system), thus allowing for an extensive array of practically relevant scenarios.

Paper Outline

In Section II we introduce our wireless network model and formulate the adaptive power control problem in arbitrarily time-varying environments using the concept of regret minimization. Our theoretical analysis is presented in Section III where we show that the FM power control scheme leads to no regret and we examine the differences that arise between the constrained and unconstrained transmit power regime. This analysis is supplemented by numerical simulations in Section IV, where we exhibit the regret minimization properties of adaptive power control in time-varying environments.

II. System Model and Problem Formulation

Our model focuses on point-to-point communication systems, where a set of *connections* $\mathcal{K} = \{1, \dots, K\}$ are established over a shared channel to connect a transmitter–receiver pair (s_k, d_k) of wireless devices. In this way, if the transmit power of transmitter s_k is denoted by p_k and the corresponding channel gain between the j-th transmitter and the d_k-th receiver is g_{kj}, the interference induced by all other users on connection k will be:

$$I_k = \sum\nolimits_{j \in \mathcal{K} \setminus \{k\}} g_{kj} p_j. \tag{1}$$

Accordingly, the signal to interference-plus-noise ratio (SINR) for connection k will be given by

$$\gamma_k(\mathbf{p}) = \frac{g_{kk} p_k}{\sigma_k^2 + I_k} = \frac{g_{kk} p_k}{\sigma_k^2 + \sum_{j \in \mathcal{K} \setminus \{k\}} g_{kj} p_j}, \tag{2}$$

where $\mathbf{p} = (p_1, \dots, p_K)$ denotes the users' power profile and σ_k^2 represents the ambient noise in connection k, including thermal, atmospheric and other peripheral interference effects (and modeled for simplicity as a complex Gaussian variable with variance σ_k^2). In this framework, the QoS requirement for connection k is that the achieved SINR must not fall below a threshold value γ^*; this amounts to the constraint

$$\gamma_k \geq \gamma_k^*, \tag{3}$$

which will often be referred to as the *feasibility constraint* of connection k.

Of course, the above model does not account for the case where the users' channel gains, their transmit powers and/or their SINR requirements vary with time – for instance, due to user mobility, fading, or adaptive bit loading that causes different constellations to have different SINR requirements and the users' target SINR to evolve over time [24]. In this case, the SINR requirement (3) for connection k takes the form

$$\gamma_k(\mathbf{p}; t) = \frac{g_{kk}(t)\, p_k}{\sigma_k^2(t) + \sum_{j \in \mathcal{K} \setminus k} g_{kj}(t)\, p_j} \geq \gamma_k^*(t), \tag{4}$$

where we have introduced an explicit dependence on t to represent the dependence of each quantity on time (assumed continuous), and we are using the notation $\gamma_k(\mathbf{p}; t)$ to emphasize that the SINR for connection k depends on both the users' transmit powers (which they control), and on the

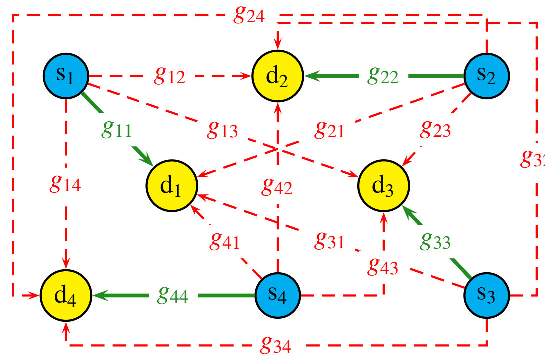

Fig. 1. Example of a network with $K = 4$ active connections (green solid lines) and the interference caused by transmitters s_k to receivers d_j, $j \neq k$ (red dashed lines). A transmitter–receiver pair (s_k, d_k), $k = 1, \dots, K$, corresponds to a wireless connection with channel gain g_{kk}; at the same time, transmitter s_j causes interference to all other receivers d_k with corresponding gain g_{kj}.

[1] We recall here that the potential divergence of power control algorithms is an important factor in the design and deployment of wireless which has been discussed extensively in the literature [7, 20–23].

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corresponding channel gain coefficients $g_{kj}(t)$ (which they do not).

In this setting, power control amounts to the time-evolving convex program:

$$\begin{aligned} \text{minimize} \quad & \textstyle\sum_{k\in\mathcal{K}} p_k \\ \text{subject to} \quad & \gamma_k(\mathbf{p};t) \geq \gamma_k^*(t), \\ & p_k \geq 0. \end{aligned} \tag{5}$$

Alternatively, the constraints of (5) can be written in linear matrix inequality (LMI) form as:

$$(\mathbf{I} - \mathbf{F}(t))\,\mathbf{p} \geq \mathbf{w}(t) \tag{6a}$$

$$\mathbf{p} \geq \mathbf{0} \tag{6b}$$

where all inequalities are to be interpreted component-wise, $\mathbf{I}$ is the identity matrix, and the quantities $\mathbf{F}(t) = (F_{kj}(t))_{k,j\in\mathcal{K}}$ and $\mathbf{w}(t) = (w_k(t))_{k\in\mathcal{K}}$ are given by

$$F_{kj}(t) = \begin{cases} 0 & \text{if } k = j, \\ \gamma_k^*(t)\,\dfrac{g_{kj}(t)}{g_{kk}(t)} & \text{otherwise,} \end{cases} \tag{7}$$

and

$$w_k(t) = \gamma_k^*(t)\,\frac{\sigma_k^2(t)}{g_{kk}(t)}. \tag{8}$$

Using this formulation, it can be shown that (5) admits a solution for $t \geq 0$ if the spectral radius $\rho(\mathbf{F}(t))$ of $\mathbf{F}(t)$ is less than 1 [5, 10, 24].

In this context, the standard Foschini–Miljanic (FM) power control dynamics take the form [5]:

$$\frac{dp_k}{dt} = -p_k\left(1 - \frac{\gamma_k^*(t)}{\gamma_k(\mathbf{p};t)}\right) \quad \forall k \in \mathcal{K}, \tag{FM}$$

where the SINR of connection k is assumed known at the transmitter s_k (e.g. through feedback). In the static regime (i.e. when channel gains, noise and target SINR levels remain constant over time), the pioneering contribution of [5] was to show that (FM) converges to the optimal solution of (5), whenever such a solution exists (i.e. whenever the problem is feasible). Otherwise, if channel gains evolve following a stationary ergodic process, the authors of [10] provided sufficient conditions for the power control dynamics (FM) to converge in distribution to a well-defined optimal configuration. However, when the problem's parameters evolve *arbitrarily* over time (and do not adhere to some stationary stochastic process with well-defined expectation), it is not known whether the power control scheme (FM) exhibits similar optimality properties.

To account for arbitrarily time-varying channels and changing SINR requirements, we will focus for simplicity on a single connection $k \in \mathcal{K}$ and we will consider the unilateral power control problem:

$$\begin{aligned} \text{minimize} \quad & p \\ \text{subject to} \quad & \gamma(p;t) \geq \gamma^*(t), \\ & p \geq 0, \end{aligned} \tag{9}$$

where we have dropped the connection index $k \in \mathcal{K}$ and we suppressed all variables that are not under the direct control of the focal transmitter, only keeping the dependence of $\gamma(p;t)$ on his transmit power p. By a simple duality argument [25], the optimization problem (9) can then be transformed into the equivalent convex problem:

$$\begin{aligned} \text{minimize} \quad & \ell(p;t) \\ \text{subject to} \quad & p \geq 0 \end{aligned} \tag{10}$$

where the *loss function*

$$\ell(p;t) = p\left(1 - \frac{\gamma^*(t)}{\gamma(p;t)}\log p\right), \tag{11}$$

represents how far the transmit power p of the focal user is from the optimal solution of (9) at time t. Indeed, using (4), a simple differentiation of (11) yields

$$\frac{\partial \ell(p;t)}{\partial p} = 1 - \frac{\gamma^*(t)}{\gamma(p;t)}, \tag{12}$$

so the Karush–Kuhn–Tucker (KKT) conditions for (10) imply that the solutions of (10) satisfy the rate constraints (3) with equality – in other words, the solution sets of (9) and (10) coincide.[2]

Obviously, since there is no direct causal link between the behavior of the focal connection and the channel gain coefficients and noise-plus-interference terms in (2), the loss function (11) may vary with time in an arbitrary fashion. In fact, the evolution of (11) over time accounts not only for changing channel gain coefficients (e.g. due to user mobility or fading), but also for time-varying QoS requirements (reflected in the target level $\gamma^*(t)$ of (11)), intermittent user activity (expressed by active and inactive summands in the interference term (1)), etc. Therefore, to incorporate as wide a range of phenomena as possible, we will assume that the user's loss function $\ell(p;t)$ evolves arbitrarily over time (owing to a combination of the above factors), our only requirement being that the stream of loss functions $t \mapsto \ell(\cdot;t)$ be locally integrable; in particular, we do not even assume continuity of $\ell(\cdot;t)$ in t.

In view of all this, we obtain the following "game against nature":

1) At each time instance t, the focal user selects a transmit power $p(t)$.
2) The user's *loss* $\ell(p(t);t)$ is determined by nature and/or the behavior of other users via (11).
3) The user updates his transmit power following (FM) and the process repeats until transmission ends.

In this context, the worst-case scenario for the user is when the environment cannot be assumed to follow some fixed probability law, so there is no Bayesian-like prior to optimize against.[3] Thus, given that static solution concepts no

[2] In particular, note that $\ell(p;t)$ grows to infinity when $p \to 0$ (representing the fact that the SINR requirements (3) are never satisfied for $p = 0$), while, for large p, $\ell(p;t)$ is asymptotically equal to the objective of the original power control problem (9).

[3] For example, in [10], the authors assume that the channel gain matrices $\mathbf{g}(t) = (g_{kj}(t))_{k,j\in\mathcal{K}}$ form a stationary ergodic sequence of random matrices, thus giving rise to a stochastic optimization problem.

longer apply in this dynamic setting, most of the literature on online optimization has focused on the criterion of *regret minimization*, a long-term optimality concept which was first introduced by Hannan [14] and which has since given rise to an extremely active field of research at the interface of optimization, statistics and theoretical computer science [15, 16].

Formally, the *cumulative regret* of the dynamic transmit policy $p(t)$ with respect to p^* is defined as:

$$\operatorname{Reg}(p^*; t) = \int_0^t \left[\ell(p(s); s) - \ell(p^*; s)\right] ds, \tag{13}$$

i.e. $\operatorname{Reg}(p^*; t)$ measures the difference between the cumulative loss incurred by a user that follows the Foschini–Miljanic dynamics (FM) and the corresponding cumulative loss incurred by the fixed transmit power p^*. The term "regret" stems from the fact that positive values of $\operatorname{Reg}(p^*; t)$ indicate that the user would have achieved his SINR requirements with lower power in the past by employing p^* instead of $p(t)$, making him "regret" his choice of transmit power at time t. Correspondingly, the user's *average regret* is defined as $t^{-1}\operatorname{Reg}(p^*; t)$ and we will say that the distributed power control scheme (FM) leads to *no regret* if

$$\limsup_{t\to\infty} \frac{1}{t}\operatorname{Reg}(p^*; t) \le 0, \tag{14}$$

for all p^* and for every possible evolution of the loss function $\ell(\cdot; t)$ over time. In words, if we interpret $\lim_{t\to\infty} t^{-1}\int_0^t p(s)\, ds$ as the long-term average power induced by the power control dynamics (FM), the no-regret criterion (14) implies that this average performs at least as well as *any* benchmark transmit power $p^* \ge 0$ in the time-varying power control problem (9).

In the static regime (where channel gains and SINR requirements are time-independent), regret minimization boils down to ordinary minimization [26], so the no-regret criterion (14) is compatible with the original convergence result of [5] for the power control dynamics (FM).[4] Furthermore, achieving non-positive regret in an online convex minimization problem is a necessary condition for tracking the minimum of the power control problem (9) as it evolves over time, so regret minimization is a natural requirement for power control under time-varying channels and/or QoS requirements. In view of the above, our main goal in what follows will be to investigate the no-regret properties of the power control dynamics (FM) in the online power minimization problem (9).

III. Power Control and Regret Minimization

In this section, our main goal will be to show that the Foschini–Miljanic power control dynamics (FM) lead to no regret with respect to the loss function (11) that measures the distance between a user's chosen transmit power and the optimal one. Our analysis will be broken up into two parts: first, we will focus on the original power control problem (9) where there are no maximum power constraints; then, to account for more realistic scenarios, we will also study a constrained variant of (9) where transmit powers cannot diverge to infinity.

[4]Similar considerations can be extended to the case of power control under ergodic channels [26].

Our first result along these lines is as follows:

Theorem 1. *For any fixed transmit power $p^* \ge 0$, the power control dynamics (FM) guarantee the cumulative regret bound:*

$$\operatorname{Reg}(p^*; t) \le R^*, \tag{15}$$

where

$$R^* = p^* \log\left[p^*/p(0)\right] + p(0) - p^*. \tag{16}$$

In particular, the power control dynamics (FM) lead to no regret against any benchmark transmit power $p^* \ge 0$.

Proof: The key observation in our proof is that thanks to (12), the power control dynamics (FM) can be written as:

$$\frac{dp}{dt} = -p\frac{\partial \ell(p; t)}{\partial p}. \tag{17}$$

Moreover, since $\ell(p; t)$ is convex with respect to p, we readily obtain

$$\ell(p(t); t) - \ell(p^*; t) \le \left[p(t) - p^*\right] \left.\frac{\partial \ell}{\partial p}\right|_{p(t)}, \tag{18}$$

and hence:

$$\begin{aligned}\operatorname{Reg}(p^*; t) &= \int_0^t \left[\ell(p(s); s) - \ell(p^*; s)\right] ds \\ &\le \int_0^t \left[p(s) - p^*\right] \left.\frac{\partial \ell}{\partial p}\right|_{p(s)} ds\end{aligned} \tag{19}$$

We thus obtain:

$$\begin{aligned}\operatorname{Reg}(p^*; t) &\le \int_0^t \left[p(s) - p^*\right]\left(-\frac{1}{p(s)}\frac{dp(s)}{ds}\right) ds \\ &= -\int_0^t \frac{dp(s)}{ds}\, ds + p^* \int_0^t \frac{1}{p(s)}\frac{dp(s)}{ds}\, ds \\ &= -\left[p(t) - p(0)\right] + p^*\left[\log p(t) - \log p(0)\right] \\ &= \left[p^* \log p(t) - p(t)\right] - \left[p^* \log p(0) - p(0)\right].\end{aligned} \tag{20}$$

Therefore, applying the standard inequality $\log(x) \le x - 1$ for $x = p(t)/p^*$, we get

$$p^* \log p(t) - p(t) \le p^* \log p^* - p^*, \tag{21}$$

and the bound (15) follows immediately. Since R^* is a constant that only depends on p^* and $p(0)$ (but not on t), we have

$$\limsup_{t\to\infty} \frac{1}{t}\operatorname{Reg}(p^*; t) \le \limsup_{t\to\infty} R^*/t = 0, \tag{22}$$

and our proof is complete. ■

Of course, even though the dynamics (FM) lead to no regret against any fixed transmit power $p^* \ge 0$, the cumulative regret guarantee (15) diverges to infinity for large p^*: more precisely, $\sup_{p^* \ge 0} R^* = +\infty$ so there is no *uniform* bound for the users' cumulative regret. As we shall see below, this is due to the fact that the power control problem (5) may be infeasible for finite p, so the power control dynamics (FM) may initiate a cascade

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of power increases that drives each user's transmit power to infinity.

The underlying problem here is that the assumption that users can transmit with effectively infinite power is unrealistic: after all, from an implementation point of view, it is impossible to supply a connection with infinite energy to support infinite power needs. As mentioned in [5], in actual situations, a connection must ultimately recognize the impossibility of co-existence with other users and leave the channel for another – or a centralized entity should intervene and lead the system to convergence by appropriate call admission control mechanisms.

In light of the above, we will modify the unilateral power control problem (9) and its loss-minimization equivalent (10) by including the power bound

$$p \leq P, \tag{23}$$

where P represents the maximum power at which the focal user can transmit (as before, the connection index $k \in \mathcal{K}$ is suppressed). Formally, this leads to the online optimization problem

$$\begin{aligned} \text{minimize} \quad & \ell(p;t) \\ \text{subject to} \quad & 0 \leq p \leq P, \end{aligned} \tag{24}$$

which now describes unilateral power control for time-varying channel conditions and/or QoS requirements under maximum transmit power constraints.

Of course, in this constrained context, the Foschini–Miljanic dynamics (FM) may lead to infeasible transmit powers $p > P$ because they do not satisfy the condition $\dot{p}(t) \leq 0$ when $p(t) = P$ (which is necessary to ensure that $p(t) \leq P$ for all $t \geq 0$). Thus, to account for (23), we will also consider the following adjusted version of the power control dynamics (FM):

$$\frac{dp(t)}{dt} = -p(t)\left(1 - \frac{p(t)}{P}\right)\left(1 - \frac{\gamma^*(t)}{\gamma(p(t);t)}\right). \tag{FM$'$}$$

Intuitively – and in contrast to (FM) – the power control dynamics (FM′) respect the constraint (23) because $\dot{p}(t) = 0$ whenever $p(t) = P$. In particular, when the user's transmit power $p(t)$ is not close to P, (FM′) essentially coincides with the original power control dynamics (FM). The difference arises when $p(t)$ approaches P: in that case, instead of simply clipping the original FM dynamics at the maximum transmit power [4], the adjusted dynamics (FM′) approach P smoothly. As such, (FM′) may be viewed as an interior-point method where the users' maximum power constraints are never saturated – though they might come arbitrarily close to that.

Our next theorem shows that the modified dynamics (FM′) lead to no regret in the online power control problem (24):

Theorem 2. *For any fixed transmit power $p^* \in [0, P]$, the adjusted power control dynamics (FM′) guarantee the cumulative regret bound:*

$$\operatorname{Reg}(p^*;t) \leq P \log 2. \tag{25}$$

In particular, the power control dynamics (FM′) lead to no regret.

Proof: As in the proof of Theorem 1, our analysis hinges on the observation that the power control dynamics (FM′) can be written in the form

$$\frac{dp}{dt} = -p\left(1 - \frac{p}{P}\right)\frac{\partial \ell(p;t)}{\partial p}. \tag{26}$$

Coupling the above with the fact that $\ell(p;t)$ is convex in p, we obtain

$$\ell(p(t);t) - \ell(p^*;t) \leq [p(t) - p^*] \left.\frac{\partial \ell}{\partial p}\right|_{p(t)} \tag{27}$$

and hence:

$$\begin{aligned} \operatorname{Reg}(p^*;t) &= \int_0^t [\ell(p(s);s) - \ell(p^*;s)]\, ds \\ &\leq \int_0^t [p(s) - p^*] \left.\frac{\partial \ell}{\partial p}\right|_{p(s)} ds. \end{aligned} \tag{28}$$

Therefore, by using (26) and rearranging, we get

$$\begin{aligned} \operatorname{Reg}(p^*;t) \leq &\left[p^* \log \frac{p(t)}{P} + (P - p^*) \log\left(1 - \frac{p(t)}{P}\right)\right] \\ &- \left[p^* \log \frac{p(0)}{P} + (P - p^*) \log\left(1 - \frac{p(0)}{P}\right)\right]. \end{aligned} \tag{29}$$

Consider now the auxiliary entropy-like function

$$h(p) = p^* \log \frac{p}{P} + (P - p^*) \log\left(1 - \frac{p}{P}\right). \tag{30}$$

By differentiating $h(p)$ with respect to p, it is easy to show that $h(p)$ is convex and attains its minimum value at $p = p^*$, i.e.

$$h(p) \geq h(p^*) = p^* \log \frac{p^*}{P} + (P - p^*) \log\left(1 - \frac{p^*}{P}\right). \tag{31}$$

Similarly, $h(p)$ attains the maximum value $h_{\max} = 0$ at the endpoints of the interval $[0, P]$. With this in mind, (29) yields the regret guarantee

$$\operatorname{Reg}(p^*;t) \leq -\left[p^* \log \frac{p^*}{P} + (P - p^*) \log\left(1 - \frac{p^*}{P}\right)\right], \tag{32}$$

and the bound (25) follows by noting that the maximum of the RHS of (32) is attained for $p^*/P = 1/2$ and is equal to $P \log 2$. The no-regret conclusion is then obtained by dividing (25) by t and taking its limit superior as $t \to \infty$. ■

Remark 1. Contrary to the regret bound (15) for unconstrained power control, the bound (25) of Theorem 2 is *uniform* and holds for any benchmark transmit power $0 \leq p^* \leq P$. As we mentioned before, this is a consequence of the fact that the system is no longer unstable (i.e. powers cannot escape to infinity). Hence, even though the users' QoS requirements may remain infeasible under the maximum transmit power constraints (23), the power control dynamics (FM′) still lead to no regret because they drive their users' transmit powers as close as possible to satisfying their QoS requirements.

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IV. Numerical Results

In order to validate the theoretical analysis of Section III, we conducted extensive numerical simulations from which we illustrate here a selection of the most representative scenarios.

For concreteness, we focused on a wireless network consisting of $K = 4$ connections with stationary transmitters and mobile receivers moving at a pedestrian speed of $v_k = 3\,\text{km/h}$. For signal transmission, we considered a shared channel with central frequency $f_c = 2.5\text{GHz}$ and sampling time $T_s = 81.6\mu\text{s}$, while channel gains were simulated following the "Pedestrian B" model with typical Jakes fading parameters [27]. For simplicity, the target SINR $\gamma_k^*(t)$ and the ambient noise $\sigma_k^2(t)$ on each connection were kept fixed over time but we set them at different values for different connections to ensure system-wide diversity of QoS requirements. Finally, for communication, we assumed a framed time division duplexing (TDD) scheme with frame duration $T_f = 5$ ms: correspondingly, transmission takes place during the downlink (DL) subframe and the SINR is fed back to transmitters during the uplink (UL) subframe; then, in the next frame, transmitters update their powers based on (FM) (or (FM′), depending on the case under consideration), and the process repeats until transmission ends.

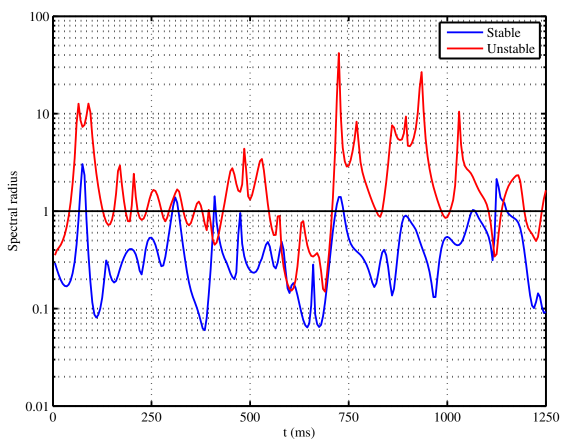


Fig. 2. Spectral radius for the examples under consideration

In this setup, we examined two different power control problems, with and without maximum power constraints. Moreover, for each of these two cases, we considered two contrasting scenarios: a relatively stable system where users can achieve their SINR requirements with finite power (most of the time at least), and an unstable network where power control is not feasible. This last dichotomy is illustrated in Fig. 2 where we depict the evolution of the spectral radius $\rho(\mathbf{F}(t))$ of the matrix (7) for the stable and unstable regime. As we noted in Section II, power control is feasible at time t if and only if $\rho(\mathbf{F}(t)) < 1$. Accordingly, a system which has $\rho(\mathbf{F}(t)) > 1$ for most of the time will be unstable: in the absence of hard, maximum power constraints, the users' transmit powers will tend to diverge to infinity as they try to meet their SINR requirements.

In the absence of maximum power constraints, the performance of the power control dynamics (FM) is illustrated in Figs. 3a–3c and 3d–3f for the stable and unstable regimes respectively. In this non-stationary, non-ergodic regime, the evolution of the power control scheme (FM) is shown in Figs. 3a and 3d: as expected, power increases in order to compensate for a channel of bad quality and decreases so as to save power when the relative channel gain is high. However, as can be seen in Figs. 3b and 3e, there is a gap between the achieved and the target SINR values (quantified by the plotted ratio $\gamma_k(p_k(t);t)/\gamma_k^*(t)$), so this adaptation is not perfect. That being said, this performance gap oscillates around zero and, despite fluctuations of great magnitude (both positive and negative), we see in Figs. 3c and 3f that the power control scheme (FM) attains the no-regret threshold very quickly, indicating that it performs asymptotically as well as the best fixed transmit power profile in hindsight. In more detail, the users' average regret starts out positive but decreases rapidly (at a rate $o(1/t)$, in accordance with Theorem 1) and soon becomes negative, thus indicating that there is no fixed power profile that could have performed better in this setting.

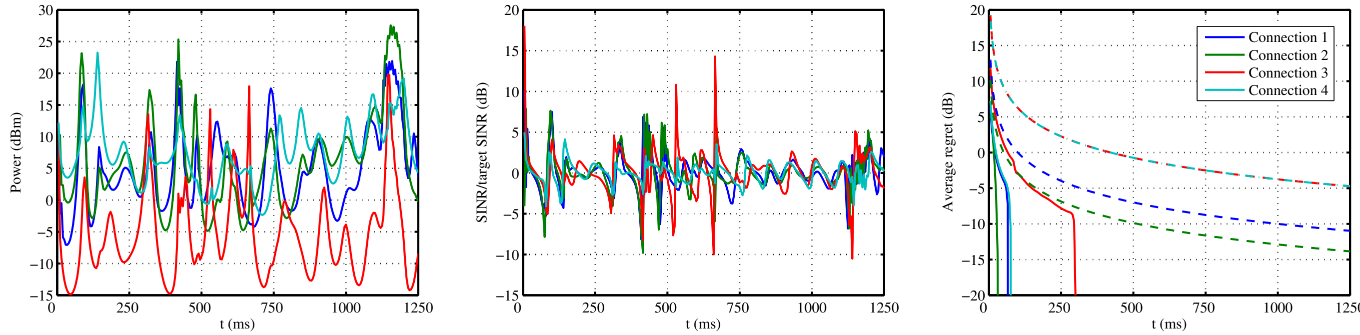
The same problem is examined in Fig. 4, but under maximum power constraints of the form $p_k \leq P_k$ (again, for both the stable and unstable regime). Qualitatively, the fundamental difference between Figs. 3 and 4 is that users now follow the adjusted power control scheme (FM′) which keeps their transmit powers below a specified, maximum value. In the stable regime, a simple comparison between Figs. 4a and 3a shows that (FM) and (FM′) behave similarly when $p_k(t) \leq P$ (and the same holds for the users' SINR gap and average regret).[5] Furthermore, we also see that the power control dynamics (FM′) allow users to approach their maximum transmit power smoothly, without clipping it abruptly.

On the other hand, in the unstable regime, the power control dynamics (FM) and (FM′) behave in a profoundly different fashion – compare Figs. 3d and 4d. In Fig. 4d, we see that two of the network's connections reach their maximum transmit power very soon, but the system retains a relatively low overall transmit power; by contrast, all transmit powers diverge to infinity in the unstable regime of Fig. 3d. Of course, the two connections whose power peaks cannot meet their QoS requirements and exhibit a negative SINR gap for almost all time (Fig. 4e). In turn, this is then reflected on the users' average regret: the connection whose power peaks after a certain time might have been *temporarily* better off if it had used a lower transmission power; however, the fact that the user's average regret tends to zero over time (cf. Fig. 4e) indicates that, *eventually*, the adjusted FM dynamics perform better than any fixed transmit power profile.

V. Conclusions

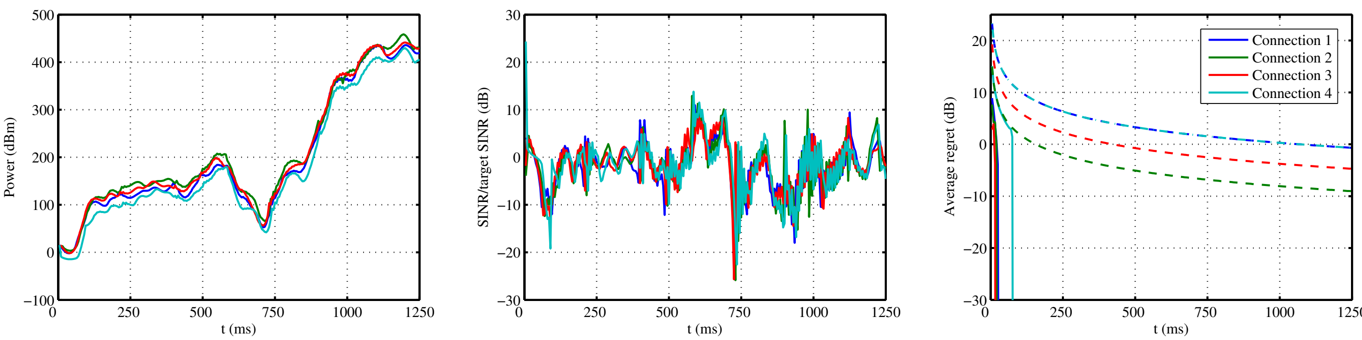
In this paper, we examined the performance of the seminal FM power control scheme in networks where channel conditions and user QoS requirements vary arbitrarily with time

[5] For comparison purposes, the simulations in Figs. 3 and 4 have been run with the same channel evolution and the same initial conditions.



(a) (b) (c)

Stable network regime: SINR levels can be achieved with finite transmit powers.



(d) (e) (f)

Unstable network regime: SINR levels cannot be achieved with finite transmit powers.

Fig. 3. Power control in the absence of maximum power constraints for a network with $K = 4$ connections in the stable and unstable regime (see legend). Figs. 3a/3d show the evolution of transmit powers based on the power control dynamics (FM), while Figs. 3b/3e show the achieved/target SINR gap $\gamma_k(p_k(t); t)/\gamma_k^*(t)$ in dB (for stable and unstable networks respectively). The users' average regret $t^{-1}\operatorname{Reg}(p^*; t)$ is plotted in Figs. 3c/3f (solid lines), along with the theoretical bounds (15) predicted by Theorem 1 (dashed lines); for simplicity, we only plot the positive part in the regret and use a logarithmic dB scale to be consistent with the transmitted power plots. For comparison purposes, we plotted the maximum regret incurred from a uniform sample of benchmark transmit powers.

(e.g. due to user mobility, fading, etc.). Contrary to the case of static and/or ergodic channels, the system's optimum power configuration evolves over time in an unpredictable fashion, so users must adapt to changes in the wireless medium (or their own requirements) "on the fly", without being able to anticipate the system's evolution.

Motivated by the theory of online optimization, we formulated the resulting power control problem as an online problem and we showed that modulo a slight adjustment in the presence of maximum power constraints, the FM power control scheme leads to no regret, i.e. it performs at least as well as (and typically outperforms) any fixed transmit profile, irrespective of how the system varies with time. For simplicity, our analysis focused on a single communication channel that evolves in continuous time; in future work, we intend to extend this analysis to several communication channels (as in an OFDMA setting) with possibly imperfect channel state information (CSI) and SINR measurements.

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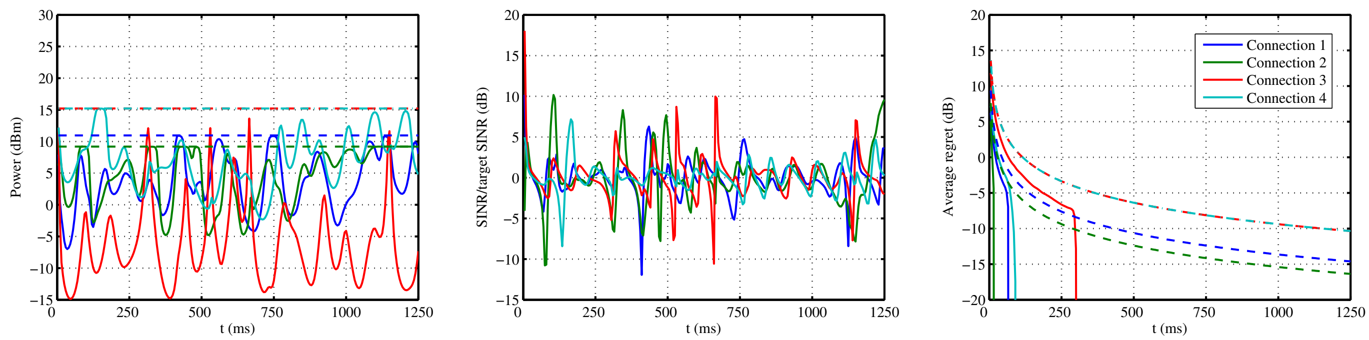
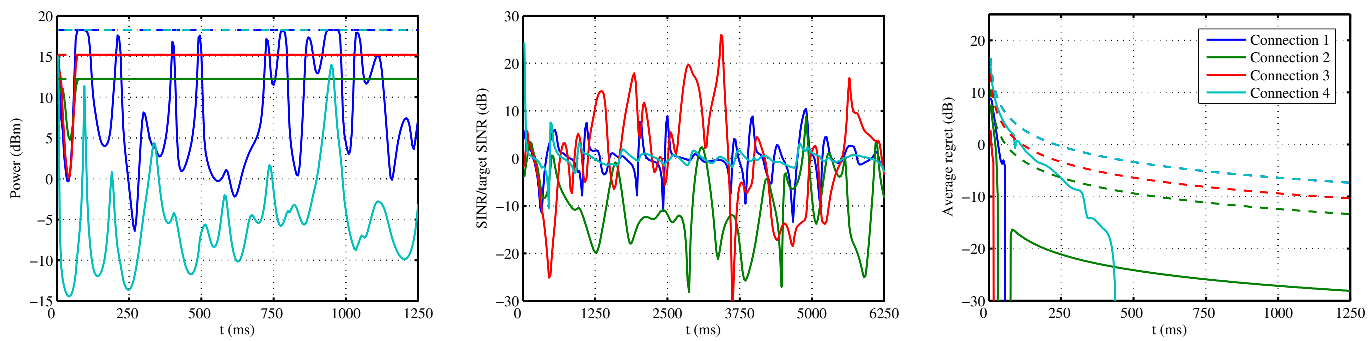


(a) (b) (c)

Stable network regime: SINR levels can be achieved with finite transmit powers.



(d) (e) (f)

Untable network regime: SINR levels cannot be achieved under the users' maximum power constraints.

Fig. 4. Power control under maximum power constraints for a network with $K = 4$ connections in the stable and unstable regime (see legend). As in Fig. 3, Figs. 4a/4d show the evolution of transmit powers based on the power control dynamics (FM′), while Figs. 4b/4e show the achieved/target SINR gap $\gamma_k(p_k(t);t)/\gamma_k^*(t)$ in dB (for stable and unstable networks respectively). The users' average regret $t^{-1}\,\mathrm{Reg}(p^*;t)$ is then plotted in Figs. 4c/4f (solid lines), along with the theoretical bounds (25) predicted by Theorem 2 (dashed lines).

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